FATIGUE TESTS AND STRESS-LIFE ($S-N$) APPROACH

- FATIGUE TESTING
  - LOADING
  - TEST MACHINES
  - SPECIMENS
  - STANDARDS

- STRESS-LIFE APPEROACH
  - $S-N$ CURVES
  - MEAN STRESS EFFECTS ON $S-N$ BEHAVIOR
  - FACTORS INFLUENCING $S-N$ BEHAVIOR
  - $S-N$ CURVE REPRESENTATION AND APPROXIMATIONS
  - EXAMPLE OF LIFE ESTIMATION USING $S-N$ APPROACH
FATIGUE LOADING

- Some load histories may be simple and repetitive, while in other cases they may be completely random.

- The randomness may contain substantial portions of more deterministic loading. For example, the ground-air-ground cycle of an aircraft has substantial similarity from flight to flight.
Ground-Air-Ground Cycle Of An Aircraft
Typical Load Histories From Actual Ground Vehicle Components
A Typical Load History of Short-Span Bridge
FATIGUE LOADING (CONTINUED)

- These load histories are typical of those found in real-life engineering situations.

- Fatigue from variable amplitude loading involving histories such as these is discussed in Chapter 9. *Constant amplitude loading* is introduced in this chapter.

- Constant amplitude loading is used:
  - To obtain material fatigue behavior/properties for use in fatigue design,
  - Some real-life load histories can occasionally be modeled as essentially constant amplitude.
CONSTANT AMPLITUDE LOADING

Minimum stress, $S_{\text{min}}$  Maximum stress, $S_{\text{max}}$  Stress range, $\Delta S$

Alternating stress, $S_a$  Mean stress, $S_m$  Stress ratio, $R$
FATIGUE LOADING (CONTINUED)

\[
S_a = \frac{\Delta S}{2} = \frac{S_{\text{max}} - S_{\text{min}}}{2}
\]

\[
S_m = \frac{S_{\text{max}} + S_{\text{min}}}{2}
\]

\[
S_{\text{max}} = S_m + S_a
\]

\[
S_{\text{min}} = S_m - S_a
\]

\[
R = \frac{S_{\text{min}}}{S_{\text{max}}}
\]

\[
A = \frac{S_a}{S_m}
\]

Stresses can be replaced with load, moment, torque, strain, deflection, or stress intensity factors.
FATIGUE LOADING (CONTINUED)

- \( R = -1 \) and \( R = 0 \) are two common reference test conditions used for obtaining fatigue properties.
  - \( R = -1 \) is called the **fully reversed** condition since \( S_{\text{min}} = -S_{\text{max}} \)
  - \( R = 0 \), where \( S_{\text{min}} = 0 \), is called **pulsating tension**.

- One **cycle** is the smallest segment of the stress versus time history which is repeated periodically.
  - Under variable amplitude loading, the definition of one cycle is not clear and hence **reversals** of stress are often considered.
  - In constant amplitude loading, one cycle equals two reversals.
FATIGUE LOADING (CONTINUED)

- Tensile and/or compressive mean loads and fully reversed loads are prevalent in all fields of engineering.

- Examples of different mean loadings.
  - The transmission history indicates significant tensile mean stress.
  - The suspension history shows significant compressive mean stress loading.
  - The bracket history is dominated by essentially fully reversed, \( R = -1 \), loading.
FATIGUE LOADING (CONTINUED)

- A thin or thick-walled pressure vessel subjected to cyclic internal pressure represents a component subjected to mean tensile stresses.

- Helical compression springs are actually under torsion, but the applied cyclic forces involve compressive mean forces.

- A cantilever beam deflected at the free end and then released to vibrate represents a damped vibration with essentially zero mean stress.
FATIGUE TEST MACHINES
Rotating Cantilever Bending Fatigue Test Machine

- Constant load amplitude
  *Non-uniform* bending moment along the specimen length
Rotating Bending Test Machine

- Constant load amplitude
  *Uniform* bending moment along the specimen length
Constant Deflection Amplitude Cantilever Bending Test Machine

- Load amplitude changes with specimen cyclic hardening or softening and decreases as cracks in the specimen nucleate and grow.

- The eccentric crank test machines do have an advantage over the rotating bending test machines in that the mean deflection, and hence the initial mean stress, can be varied.
Schematic of an Axial Loaded Fatigue Test Machine

Capable of applying both mean and alternating axial loads in tension and/or compression
A Test Setup For Combined In-phase Torsion and Bending With or Without Mean Stress

Uniform torque and a non-uniform bending moment along the specimen length
A Modern Servo-hydraulic Test System

- **Principle of operation includes:**
  - generating an input signal of load, strain, or displacement using a function generator,
  - applying this input through a hydraulic actuator, measure the specimen response via a load cell, clip gage, or an LVDT,
  - compare this output with the input. The difference drives the system.

- Control and test data outputs are usually through a PC and software.
- Test frequency can range from mHz to kHz.
A Modern Servo-hydraulic Test System

- These test systems can perform:
  - constant or variable amplitude load, strain, deformation, or stress intensity factor controlled tests on small specimens or
  - can be utilized with hydraulic jacks for components, subassemblies, or whole structures.

- Two or more control systems are used for multiaxial testing.
COMMON FATIGUE TEST SPECIMENS
(a) Rotating bending, (b) Axial uniform, (c) Axial hourglass

- These specimens are usually used for axial or bending tests.
- These specimens usually have finely polished surfaces to minimize surface roughness effects.
- No distinction between crack nucleation and growth is normally made with these specimens.
- Careful alignment is needed for axial loaded specimens to minimize bending.
COMMON FATIGUE TEST SPECIMENS
(Axial or bending with circumferential groove)

- Stress concentration influence can be studied with most of these specimens by machining in notches, holes, or grooves.
COMMON FATIGUE TEST SPECIMENS

- A *thin-walled tube* specimen designed for torsion and combined axial/torsion with the possibility of adding internal and/or external pressure.
  - This multiaxial loading can be performed in-phase or out-of-phase.
  - The thin-walled tube allows for essentially uniform normal and shear stresses in the cross-sectional area.
Cantilever Flat Sheet Specimen
COMMON FATIGUE TEST SPECIMENS

- Specimens shown have been used for obtaining *fatigue crack growth* data.
- In all cases a thin slit, notch, or groove with a very small root radius is machined into the specimen.
- Fatigue crack growth testing is covered in Section 6.4.
ASTM Standard Practices Related to Fatigue Testing of Metals

- **E466** Conducting Force Controlled Constant Amplitude Axial Fatigue Tests of Metallic Materials.

- **E467** Verification of Constant Amplitude Dynamic Forces in an Axial Fatigue Testing System.

- **E468** Presentation of Constant Amplitude Fatigue Test Results for Metallic Materials.

- **E606** Strain-Controlled Fatigue Testing.
ASTM Standard Practices Related to Fatigue Testing of Metals (Continued)

- **E647** Measurement of **Fatigue Crack Growth Rates**.

- **E739** **Statistical Analysis** of Linear or Linearized Stress-Life ($S-N$) and Strain-Life ($\varepsilon-N$) Fatigue Data.

- **E1012** Verification of **Specimen Alignment** Under Tensile Loading

- **E1049** **Cycle Counting** in Fatigue Analysis.

- **E1823** Standard **Terminology** Relating to Fatigue and Fracture Testing.
ISO Standards Related to Fatigue Testing of Metals


- ISO/DIS 12107  Metallic Materials-Fatigue Testing-**Statistical Planning and Analysis of Data**.

STRESS-LIFE (S-N) APPROACH

- S-N CURVES
- MEAN STRESS EFFECTS ON S-N BEHAVIOR
- FACTORS INFLUENCING S-N BEHAVIOR
- S-N CURVE REPRESENTATION AND APPROXIMATIONS
- EXAMPLE OF LIFE ESTIMATION USING S-N APPROACH
STRESS-LIFE CURVES, $S-N$

- Typical schematic $S-N$ curve obtained under axial load or stress control test conditions with smooth specimens.

- Constant amplitude $S-N$ curves of this type are plotted on semi-log or log-log coordinates.
STRESS-LIFE CURVES, $S-N$

- $S-N$ curves obtained under torsion or bending load-control test conditions often do not have data at the shorter fatigue lives (say $10^3$ or $10^4$ cycles and less) due to significant plastic deformation.

- Torsion and bending stress equations $\tau = Tr/J$ and $\sigma = My/I$ can only be used for nominal elastic behavior.
Typical variability with less variability at shorter lives and greater variability at longer lives.

Variability in life for a given stress level can range from less than a factor of two to more than an order of magnitude.

Variability and statistical aspects of fatigue data are discussed in Ch. 13.
Fig. (a) shows a continuous sloping curve, while Fig. (b) shows a discontinuity or “knee” in the $S-N$ curve.

- This knee has been found in only a few materials (i.e. low and medium strength steels) between $10^6$ and $10^7$ cycles in non-corrosive conditions.
- Most materials do not contain the “knee” even under controlled environments.
- Under corrosive environments all $S-N$ data have a continuous sloping curve.

When sufficient data are available, $S-N$ curves are usually drawn through median points and thus represent 50 percent expected failures.
Common terms used with the $S-N$ diagram are

- **Fatigue life, $N_f$:** The number of cycles of stress or strain that a specimen sustains before failure occurs.

- **Fatigue strength:** A hypothetical value of stress at failure for exactly $N_f$ cycles as determined from an $S-N$ diagram.

- **Fatigue limit, $S_f$:** The limiting value of the median fatigue strength as $N_f$ becomes very large. Endurance limit is often implied as being analogous to the fatigue limit.
Fatigue typically consists of crack nucleation, growth, and final fracture, as it was emphasized in Chapter 3.

A reasonable crack nucleation life can be defined by a crack length of 0.25mm (0.01in.). This dimension can relate to engineering dimensions and can represent a small macrocrack.

The number of cycles to form this small crack in smooth unnotched or notched fatigue specimens and components can range from a few percent to almost the entire life, as illustrated schematically.
A larger fraction of life for crack growth, the shaded area, occurs at higher stress levels, while a larger fraction of life for crack nucleation occurs at lower stress levels.

When fatigue crack growth life is significant, then fracture mechanics, as discussed in Chapter 6, should be used.
STRESS-LIFE CURVES, $S-N$ (Continued)

- The **fatigue limit** has historically been a prime consideration for long-life fatigue design.

- For a given material the fatigue limit has an enormous range depending on:
  - surface finish,
  - size,
  - type of loading,
  - temperature,
  - corrosive, and other aggressive environments,
  - mean stresses,
  - residual stresses, and
  - stress concentrations.
Fatigue limit based on a nominal alternating stress, $S_a$:

- Can range from essentially 1 to 70 percent of the ultimate tensile strength.
- Example of a case where the fatigue limit may be approximately 1 percent of $S_u$ is a high strength steel with a sharp notch subjected to a high mean tensile stress in a very corrosive atmosphere.
- An example of a case when the fatigue limit might approach 70 percent of $S_u$ is a medium strength steel in an inert atmosphere containing appreciable compressive residual stresses.
Most long-life S-N fatigue data available in the literature consist of **fully reversed** \( (S_m = 0) \) **uniaxial** fatigue strengths or fatigue limits of **small highly polished unnotched** specimens based on \( 10^6 \) to \( 5 \times 10^8 \) cycles to failure in **laboratory air environment**.

Representative monotonic tensile properties and bending fatigue limits of selected engineering alloys obtained under the above conditions are given in **Table A.1**.

- The fatigue limits given in Table A.1 must be substantially reduced in most cases before they can be used in design situations.
- For example, 10 to 25 percent reductions in these values for just size effect alone is not unreasonable for bend specimens greater than 10 mm (0.4 in.) in diameter.
<table>
<thead>
<tr>
<th>Material</th>
<th>Process Description</th>
<th>Hardness</th>
<th>$S_u$ (MPa/ksi)</th>
<th>$S_y$ (MPa/ksi)</th>
<th>%EI</th>
<th>%RA</th>
<th>$S_n$ (MPa/ksi)</th>
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<td>503(73)</td>
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<td>158(23)</td>
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</tbody>
</table>

Others

| Ti6Al-4V | Annealed            | 520(75)  | 330(52)        |
| Copper   | Annealed            | 315(46)  | 85(12)         |
| Phosphor Bronze | Annealed  | 340(49)  | 170(25)        |

* Incomplete information on surface finish. These values do not represent design fatigue limits.
Rotating Bending Fatigue Limits or Fatigue Strengths Based on $10^7$ to $10^8$ Cycles for Steels
Rotating Bending Fatigue Limits or Fatigue Strengths Based on $10^7$ Cycles for **Irons**

- Nodular cast iron
- Ingot iron
- Malleable cast iron
- Wrought iron
Rotating Bending Fatigue Limits or Fatigue Strengths Based on $10^8$ Cycles for **Aluminum Alloys**

* cast  x wrought
Rotating Bending Fatigue Limits or Fatigue Strengths for **Wrought Copper Alloys**
STRESS-LIFE CURVES, $S-N$ (Continued)

- $S_f/S_u$ varies from about 0.25 to 0.65 for these data.

- There is a tendency to generalize that $S_f$ increases linearly with $S_u$. These figures show this is incorrect and data bands tend to bend over at the higher ultimate strengths.
For steels, substantial data are clustered near fatigue ratio $S_f/S_u \approx 0.5$ for the low and medium strength steels.

- The data actually fall between 0.35 and 0.6 for $S_u < 1400$ MPa (200 ksi).
- For $S_u > 1400$ MPa (200 ksi), $S_f$ does not increase significantly.

Common estimates for unnotched, highly polished, small bending specimen fatigue limits for steels are:

- $S_f = 0.5 S_u$ for $S_u \leq 1400$ MPa (200 ksi)
- $S_f = 700$ MPa for $S_u \geq 1400$ MPa (200 ksi)

For steels, $S_u$ can be approximated from the Brinell hardness, $HB$, as:

- $S_u = 3.45 HB$ for MPa units
- $S_u = 0.5 HB$ for ksi units
These equations are not unreasonable for *small highly polished steel specimens*, but empirical reduction factors for surface finish, size, stress concentration, temperature, and corrosion must also be considered.

We strongly warn against using a design fatigue limit equal to one-half the ultimate strength for steels. Most data in Fig. 4.8 for irons, aluminum, and copper alloys fall below the 0.5 fatigue ratio.

The aluminum and copper alloy data bands bend over at higher strengths as do the bands for steels. Thus high strength steels, aluminum, and copper alloys generally do not exhibit corresponding high unnotched fatigue limits.
Schematic Scatter Bands For Steels
MEAN STRESS EFFECTS ON S-N BEHAVIOR

- The mean stress, $S_m$, can have substantial influence on fatigue behavior.

- In general, tensile mean stresses are detrimental and compressive mean stresses are beneficial.
MEAN STRESS EFFECTS ON S-N BEHAVIOR

- At intermediate or high stress levels under load control test conditions, substantial **cyclic creep** (also referred to as **cyclic ratcheting**) which increases the mean strain, can occur in the presence of mean stresses.

- This **cyclic creep** adds to the detrimental effects of tensile mean stress on fatigue life and results in additional undesirable excess deformation.
Tensile Mean Stress Influence on Long-life ($10^7$ Cycles) Fatigue Strength For Steel Alloys

- $S_f$ is the fully reversed, ($S_m = 0, R = -1$), fatigue limit of smooth specimens
- Similar behavior exists for other alloys.
- The general trend indicates that tensile mean stresses are detrimental.
- Much of the data fall between the straight and curved lines.
Tensile Mean Stress Influence on Long-life ($5 \times 10^7$ Cycles) Fatigue Strength For Aluminum Alloys
MEAN STRESS EFFECTS ON S-N BEHAVIOR (CONT’D)

- The straight line is the **modified Goodman** line.

\[
\frac{S_a}{S_f} + \frac{S_m}{S_u} = 1
\]

- The curve is the **Gerber** parabola.

\[
\frac{S_a}{S_f} + \left(\frac{S_m}{S_u}\right)^2 = 1
\]

- An additional popular relationship has been formulated by replacing \(S_u\) with \(\sigma_f\) (**Morrow** line) where \(\sigma_f\) is the true fracture strength.

\[
\frac{S_a}{S_f} + \frac{S_m}{\sigma_f} = 1
\]
Compressive Mean Stress Effects for Several Steels and Aluminum Alloys

- These compressive mean stresses cause increases of up to 50 percent in the alternating fatigue strength.

- This increase is too often overlooked, since compressive residual stresses can cause similar beneficial behavior.

- The modified Goodman or Morrow equations can be extrapolated into the compression mean stress region.

- The Gerber equation incorrectly predicts a detrimental effect of compressive mean stresses.

Figure 4.13: Compressive and tensile mean stress effect [12]. (●) Aluminum alloys, (○) steels.
MEAN STRESS EFFECTS ON S-N BEHAVIOR (CONT’D)

- The modified Goodman and Morrow equations are shown for a given long life (e.g. 10⁷ cycles) along with the criterion for yielding:

\[ \frac{S_a}{S_y'} + \frac{S_m}{S_y} = 1 \]

- If the coordinates of the applied alternating and mean stresses fall within the modified Goodman or Morrow lines, then fatigue failure should not occur prior to the given life.
MEAN STRESS EFFECTS ON S-N BEHAVIOR (CONT’D)

- If **yielding** is not to occur, then the applied alternating and mean stresses must fall within the two yield lines connecting $\pm S_y$ to $S'_y$.

- If both fatigue failure and yielding are not to occur, then neither criterion, as indicated by the three bold lines should be exceeded.
FACTORS INFLUENCING S-N BEHAVIOR

- The reference fatigue condition for S-N behavior is usually fully reversed $R = -1$ bending or axial loading using small unnotched specimens.

- In addition to mean stress many other factors also affect the reference fatigue condition.

- Some of these are discussed in other chapters:
  - notches and stress concentrations (Chapter 7)
  - residual stress and surface treatment (Chapter 8)
  - variable amplitude loading (Chapter 9)
  - multiaxial and torsion loading (Chapter 10)
  - corrosion (Section 11.1)
  - fretting (Section 11.2)
  - low temperature (Section 11.3)
  - high temperature (Section 11.4)
FACTORS INFLUENCING S-N BEHAVIOR (CONT’D)

- Additional factors that influence S-N behavior are:
  - Microstructure
  - Size Effects
  - Surface Finish
  - Frequency
FACTORS INFLUENCING S-N BEHAVIOR (MICROSTRUCTURE)

- Metal fatigue is significantly influenced by microstructure.

- Microstructure includes:
  - chemistry,
  - heat treatment,
  - cold working,
  - grain size,
  - anisotropy,
  - inclusions,
  - voids/porosity, and
  - other discontinuities or imperfections.
FACTORS INFLUENCING S-N BEHAVIOR (MICROSTRUCTURE, CONT’D)

- If the actual S-N data are available, microstructural effects are inherently accounted for and do not have to be accounted for again.

- Chemistry, heat treatment, and cold working have an enormous number of synergistic variations, and generalities concerning their effects on fatigue behavior can not be made.
FACTORS INFLUENCING $S-N$ BEHAVIOR
(MICROSTRUCTURE, CONT’D)

- Some generalities for the other microstructural aspects.
  - **Fine grain size** generally provides better $S-N$ fatigue resistance than coarser grains, except at elevated temperatures where creep/fatigue interaction exists.
  - **Fine grains** reduce localized strains along slip bands reducing the amount of irreversible slip and provide more grain boundaries to aid in transcrystalline crack arrest and deflection, and thus reduce fatigue crack growth rates.
FACTORS INFLUENCING S-N BEHAVIOR (MICROSTRUCTURE, CONT’D)

- **Anisotropy** caused by cold working gives increased S-N fatigue resistance when loaded in the direction of the working than when loaded in the transverse direction. This is due to the elongated grain structure in the direction of the cold working.

- **Inclusions, and voids/porosity** act as stress concentrations and thus are common locations for microcracks to nucleate under cyclic load, or to form during heat treatment or cold working prior to cyclic loading. Minimizing inclusions, voids/porosity, and other discontinuities through carefully controlled production and manufacturing procedures is a key to good fatigue resistance.
FACTORS INFLUENCING S-N BEHAVIOR (SIZE EFFECTS)

- Under unnotched bending conditions if the diameter or thickness of the specimen is < 10mm (0.4 in.) then the $S-N$ fatigue behavior for steels is reasonably independent of the diameter or thickness.

- For larger size, the $S-N$ fatigue resistance is decreased as the diameter or thickness increases to 50 mm (2 in.), the fatigue limit for steels decreases to a limiting factor of about 0.7 to 0.8 of the fatigue limit for specimens less than 10 mm (0.4 in.) in diameter or thickness.

- Additional decreases can occur for larger specimens or components.

- Under unnotched axial conditions the $S-N$ fatigue resistance is poorer than for most bending conditions. The fatigue limit for axial loading can be from 0.75 to 0.9 of the small specimen bending fatigue limits.
Several factors are involved in size and axial loading effects.

- In **bending**, the larger the diameter or thickness, the smaller the **bending stress gradient** and hence the larger the average stress in a local region on the surface. The average stress in the local region may be the governing stress for fatigue rather than the maximum stress.

- For **axial loaded** unnotched specimens, a nominal stress gradient does not exist, and the average and maximum nominal stresses have the same magnitude resulting in less size effect than in bending.
FACTORS INFLUENCING S-N BEHAVIOR (SIZE EFFECTS)

- In bending and axial loading, larger specimens have a higher **probability of microstructural discontinuity density** in the highly stressed surface regions that contribute to the decrease in fatigue resistance.

- Another reason why axial fatigue resistance is lower than in bending is possible **eccentricity** or alignment difficulties that superimpose bending stresses on the axial stresses.
FACTORS INFLUENCING S-N BEHAVIOR
(SURFACE FINISH EFFECTS)

- Since most fatigue failures originate at the surface, the surface will have a substantial influence on fatigue behavior.

- Surface effects are caused by differences in surface roughness, microstructure, chemical composition, and residual stress.

- This influence will be more pronounced at long lives where a greater percentage of the cycles is usually involved with crack nucleation.

- Reference fatigue strengths are for highly polished smooth specimens. Most engineering parts, however, are not highly polished and grinding or machining, will cause degradation in fatigue strength.
Surface Factors, $K_s$, as a Function of Ultimate Tensile Strength For Steels

- The higher the ultimate tensile strength and hardness, the greater the degradation of fatigue limits.

- The decreases caused from grinding and machining are more related to surface roughness and residual stresses, while hot-rolled and as-forged behavior include these two important aspects along with surface microstructural and chemical composition changes such as decarburization and hence surface hardenability.

- Avoid the hot-rolled or as-forged surface conditions at fatigue sensitive locations by removing the undesirable surface by grinding or machining.
FACTORS INFLUENCING S-N BEHAVIOR (FREQUENCY EFFECTS)

- The influence of frequency on S-N behavior of metals is complicated because of synergistic effects of test temperature, corrosive environment, stress-strain sensitivity to strain rate, and frequency.

- Specimen heating at higher test frequency due to internal hysteresis damping can increase the specimen temperature and thus disguise the true ambient temperature fatigue behavior.

- Generation of heat due to cyclic loading depends on the volume of highly stressed material. Axial loading will produce more heat than bending or notched specimens. Thus, frequency effects can be different.

- If heating and corrosion effects are negligible, frequencies from less than 1 Hz to 200 Hz have had only a small effect on S-N behavior for most structural metals.
**S-N CURVE REPRESENTATION AND APPROXIMATIONS**

- Actual fatigue data from either specimens or parts should be used in design, if possible.

- Oftentimes this information is not available and must either be generated or approximations of \( S-N \) behavior must be made.

- Common reasonable \( S-N \) median fatigue life curves based upon straight-line log-log approximations are shown.
Common Reasonable $S-N$ Approximations
**S-N Curve Representation and Approximations (Continued)**

- **Basquin** suggested a *log-log straight line S-N relationship* such that

\[ S_a \text{ or } S_{Nf} = A \left( N_f \right)^B \]

- \( S_a \) is an applied alternating stress
- \( S_{Nf} \) is the fully reversed, \( R = -1 \), fatigue strength at \( N_f \) cycles
- \( A \) is the coefficient and represents the value of \( S_a \) or \( S_{Nf} \) at one cycle
- \( B \) is the slope of the log-log S-N curve
One approximate representation of $S-N$ curve is a **tri-slope model** with one slope between one cycle and $10^3$ cycles, one between $10^3$ and $10^6$ or $10^8$ cycles, and another slope after $10^6$ or $10^8$ cycles.

- The tri-slope model indicates a fatigue limit does not exist, which may be the case for service variable amplitude loading.
- The tri-slope model exists in some design codes such as for gears and welds.
- The tri-slope model could also have the third, or long-life, slope be horizontal after $10^6$ or $10^8$ cycles.
Other approximation models assume one sloping straight line from one cycle to $10^6$, $10^7$, or $10^8$ cycles followed by a horizontal line or another sloped line.

The intercept, $A$, at $N_f = 1$ could be chosen as the ultimate tensile strength, $S_u$, the true fracture strength, $\sigma_f$, or fatigue strength coefficient, $\sigma_f'$.

Basquin’s equation using $\sigma_f'$ is based on reversals, $2N_f$, rather than cycles, $N_f$.

\[ S_a \text{ or } S_{Nf} = \sigma_f'(2N_f)^b \]

Values of the $\sigma_f'$ and $b$, defined as the fatigue strength coefficient and the fatigue strength exponent respectively, are given in Table A.2 for a few selected engineering alloys.
<table>
<thead>
<tr>
<th>Material</th>
<th>Process Description</th>
<th>$S_u$ (MPa)</th>
<th>E (GPa)</th>
<th>$S_{y}/S_{y}^{'}$</th>
<th>K/K'</th>
<th>n/n'</th>
<th>$\sigma_{r}/\sigma_{r}^{'}$</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>HR sheet</td>
<td>331 (48)</td>
<td>203</td>
<td>80</td>
<td>534/867</td>
<td>0.185/0.244</td>
<td>1.63/0.104</td>
<td>4.99</td>
<td>-0.005/0.048</td>
</tr>
<tr>
<td></td>
<td>HR sheet</td>
<td>441 (64)</td>
<td>203</td>
<td>62</td>
<td>738/1962</td>
<td>0.190/0.321</td>
<td>0.96/0.337</td>
<td>1384</td>
<td>-0.156/0.485</td>
</tr>
<tr>
<td></td>
<td>Normalized</td>
<td>582 (84)</td>
<td>201</td>
<td>54</td>
<td>1106/1340</td>
<td>0.259/0.220</td>
<td>0.77/0.309</td>
<td>898/1043</td>
<td>-0.107/0.481</td>
</tr>
<tr>
<td></td>
<td>Q &amp; T</td>
<td>649 (94)</td>
<td>219</td>
<td>67</td>
<td>1183/1330</td>
<td>0.221/0.208</td>
<td>1.10/0.255</td>
<td>1197/1009</td>
<td>-0.097/0.460</td>
</tr>
<tr>
<td>Man-Ten</td>
<td>HR sheet</td>
<td>510 (74)</td>
<td>207</td>
<td>64</td>
<td>393/372</td>
<td>0.20/0.11</td>
<td>1.02/0.86</td>
<td>814/807</td>
<td>-0.071/0.65</td>
</tr>
<tr>
<td>RQC-100</td>
<td>HR sheet</td>
<td>931 (135)</td>
<td>207</td>
<td>64</td>
<td>883/600</td>
<td>0.06/0.14</td>
<td>1.02/0.66</td>
<td>1330/1240</td>
<td>-0.07/0.69</td>
</tr>
<tr>
<td></td>
<td>Annealed</td>
<td>752 (109)</td>
<td>225</td>
<td>44</td>
<td>517/—</td>
<td>0/0.152</td>
<td>0.58/0.486</td>
<td>916/—</td>
<td>-0.079/0.520</td>
</tr>
<tr>
<td></td>
<td>Q &amp; T</td>
<td>1827 (265)</td>
<td>500</td>
<td>51</td>
<td>1689/—</td>
<td>0.047/0.145</td>
<td>0.71/0.196</td>
<td>2661/—</td>
<td>-0.093/0.643</td>
</tr>
<tr>
<td></td>
<td>Normalized</td>
<td>1090 (158)</td>
<td>259</td>
<td>14</td>
<td>735/545</td>
<td>0.158/0.174</td>
<td>0.15/0.250</td>
<td>1310/—</td>
<td>-0.091/0.496</td>
</tr>
<tr>
<td></td>
<td>Q &amp; T</td>
<td>1147 (166)</td>
<td>309</td>
<td>22</td>
<td>650/627</td>
<td>0.165/0.176</td>
<td>0.24/0.700</td>
<td>1878/—</td>
<td>-0.120/0.600</td>
</tr>
<tr>
<td></td>
<td>Normalized</td>
<td>789 (115)</td>
<td>229</td>
<td>47</td>
<td>493/481</td>
<td>0.187/0.177</td>
<td>0.64/0.602</td>
<td>1117/1326</td>
<td>-0.103/0.581</td>
</tr>
<tr>
<td></td>
<td>Q &amp; T</td>
<td>925 (134)</td>
<td>277</td>
<td>59</td>
<td>814/591</td>
<td>0.074/0.124</td>
<td>0.88/0.309</td>
<td>1405/1127</td>
<td>-0.066/0.514</td>
</tr>
<tr>
<td></td>
<td>Q &amp; T</td>
<td>1413 (205)</td>
<td>380</td>
<td>48</td>
<td>1378/—</td>
<td>0.051/0.124</td>
<td>0.65/0.637</td>
<td>2143/—</td>
<td>-0.094/0.761</td>
</tr>
<tr>
<td></td>
<td>Q &amp; T</td>
<td>1929 (280)</td>
<td>475</td>
<td>35</td>
<td>1722/—</td>
<td>0.048/0.094</td>
<td>0.43/0.331</td>
<td>2161/—</td>
<td>-0.081/0.854</td>
</tr>
<tr>
<td>4340</td>
<td>HR</td>
<td>827 (120)</td>
<td>243</td>
<td>43</td>
<td>634/—</td>
<td>0.168/0.57/0.522</td>
<td>1198/—</td>
<td>-0.095/0.563</td>
<td></td>
</tr>
</tbody>
</table>
The slope, $B$, depends upon many factors and for unnotched parts could vary from about $-0.05$ to $-0.2$.

For example, the varying slope can be indicated by using surface effects with $S_f$ taken from Fig. 4.15.

- Surface effects are dominant at long fatigue lives and less significant at short lives with convergence of the $S-N$ curves at $S_u$ at $N_f = 1$.

- This gives different values of slope $B$ for each surface condition.

- The convergence at $N_f = 1$ is reasonable because surface finish does not have an appreciable affect on monotonic properties for most smooth metal specimens.
Oftentimes the slope, $B$, for smooth unnotched specimens is about $-0.1$.

- This suggests that for unnotched specimens the fatigue life is approximately inversely proportional to the $10^{th}$ power of alternating stress.
- Thus, a $10\%$ increase or decrease in alternating stress will cause about a factor of three decrease or increase, respectively, in fatigue life.
- For notched parts the slope of the $S-N$ curve on logarithmic scales is steeper yielding more extreme changes.
- Thus, even small changes in applied alternating stress can have a significant effect on fatigue life.
Figure 4.17  Constant life diagrams with superimposed yield criterion.
The following equations provide information to determine estimates of allowable $S_a$ and $S_m$ for a given fatigue life of unnotched parts.

\[ S_{Nf} = A \left( N_f \right)^B \]

and

\[ \frac{S_a}{S_{Nf}} + \frac{S_m}{S_u} = 1 \quad \text{or} \quad \frac{S_a}{S_{Nf}} + \frac{S_m}{\sigma_f} = 1 \]
SUMMARY AND DOS AND DON’TS IN DESIGN

- **Test systems** are available to perform fatigue and durability tests for almost every conceivable situation from a small highly polished laboratory specimen to that of a large scale complex structure.

- The **fatigue limit** under constant amplitude loading conditions occurs for a few metals (notably low and medium strength steels), but under in-service variable amplitude loading with corrosive, temperature, or other environmental conditions, the fatigue limit is rare.

- The fully reversed rotating beam smooth specimen fatigue strength, $S_f$, at $10^6$ to $10^8$ cycles ranges from about 0.25 to 0.65 times the ultimate tensile strength, $S_u$. For real parts, this can vary from about 0.01 to 0.7.
SUMMARY AND DOS AND DON’TS IN DESIGN (CONT’D)

- Yielding, finite or long life, and mean stress effects can be approximated with the following models:

Yielding:

\[ \frac{S_a}{S_y'} + \frac{S_m}{S_y} = 1 \]

Basquin equation:

\[ S_{Nf} = A \left( N_f \right)^B \]

Mean stress finite life

\[ \frac{S_a}{S_{Nf}} + \frac{S_m}{S_u} = 1 \quad \text{or} \quad \frac{S_a}{S_{Nf}} + \frac{S_m}{\sigma_f} = 1 \]

Mean stress long life

\[ \frac{S_a}{S_f} + \frac{S_m}{S_u} = 1 \quad \text{or} \quad \frac{S_a}{S_f} + \frac{S_m}{\sigma_f} = 1 \]
SUMMARY AND DOS AND DON’TS IN DESIGN (CONT’D)

- Do consider that the fully reversed fatigue strength, $S_{fr}$, at $10^6$ to $10^8$ cycles for components can vary from about 1 to 70 percent of the ultimate tensile strength.

- Do note that cleaner metals, and generally smaller grain size for ambient temperature, have better fatigue resistance.

- Do recognize that frequency effects are generally small only when corrosion, temperature or other aggressive environmental effects are absent.
SUMMARY AND DOS AND DON’TS IN DESIGN (CONT’D)

- Do consider that **surface finish** can have a substantial influence on fatigue resistance particularly at longer lives.

- Don't neglect the advantages of compressive mean or compressive residual stresses in improving fatigue life and the detrimental effect of tensile mean or tensile residual stresses in decreasing fatigue life.

- Do attempt to use actual fatigue data in design, but if this is not possible or reasonable, approximate estimates of median fatigue behavior can be made.
EXAMPLE PROBLEM USING STRESS-LIFE (S-N) APPROACH