Advances in Fatigue and Fracture Mechanics

June 2nd – 6th, 2014,

Aalto University,

Espoo, Finland

given by

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University of Illinois, USA

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University of Waterloo, Canada

Bibliography

1. Bannantine, J., Corner, Handrock**, <u>Fundamentals of Metal Fatigue Analysis</u>, Prentice-Hall, 1990....** (good general reference)

2. Dowling, N., **<u>Mechanical Behaviour of Materials</u>, Prentice Hall, 2011, 3rd edition** (middle chapters are a great overview of most recent approaches to fatigue analysis),

3. Stephens, R.I., Fatemi, A., Stephens, A.A., Fuchs, H.O., **<u>Metal Fatigue in Engineering</u>, John Wiley, 2001....** (good general reference),

4. M. Janssen, J. Zudeima, R.J.H. Wanhill**, <u>Fracture Mechanics</u>, VSSD, The Netherlands, 2006** (understandable, rigorous, mechanics perspective),

5. Socie, D.F., and Marquis, G.B., **<u>Multiaxial Fatigue</u>**, Society of Automotive Engineers, Inc., Warrendale, PA, 2000

5. Haibach, E., **<u>Betriebsfestigkeit</u>**, VDI Verlag, Dusseldorf, 1989 (in German).

6. Bathias, C., and Pineau, A., **<u>Fatigue des Materiaux et des Structures</u>**, Hermes, Paris, 2008 (in French and English),

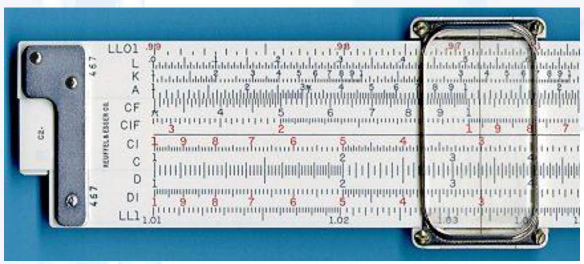
7. Radaj, D., **<u>Design and Analysis of Fatigue Resistant Structures</u>, Halsted Press, 1990,** (Complete, civil and automotive engineering analysis perspective),

8. V.A. Ryakhin and G.N. Moshkarev**, <u>Durability and Stability of Welded Structures in Earth Moving Machinery", Mashinostroenie</u>,** Moscow, 1984 (in Russian, cranes and earth moving machinery),

9. A. Chattopadhyay, G. Glinka, M. El-Zein, J. Qian and R. Formas, Stress Analysis and Fatigue of Welded Structures, **<u>Welding in the World</u>**, (IIW), vol. 55, No. 7-8, 2011, pp. 2-21.

Mechanical Engineer – yesterday and today……

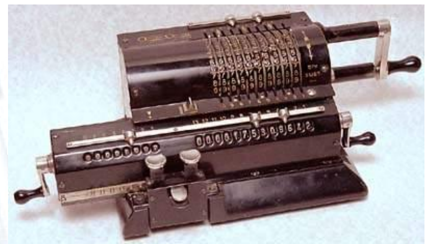
Slide ruler



≈ 1-2 operation/min.

Before yesterday

Calculator



≈ 1-10 operation/min.

Yesterday

Computer PC/laptop

≈ 10[?] operation/min.

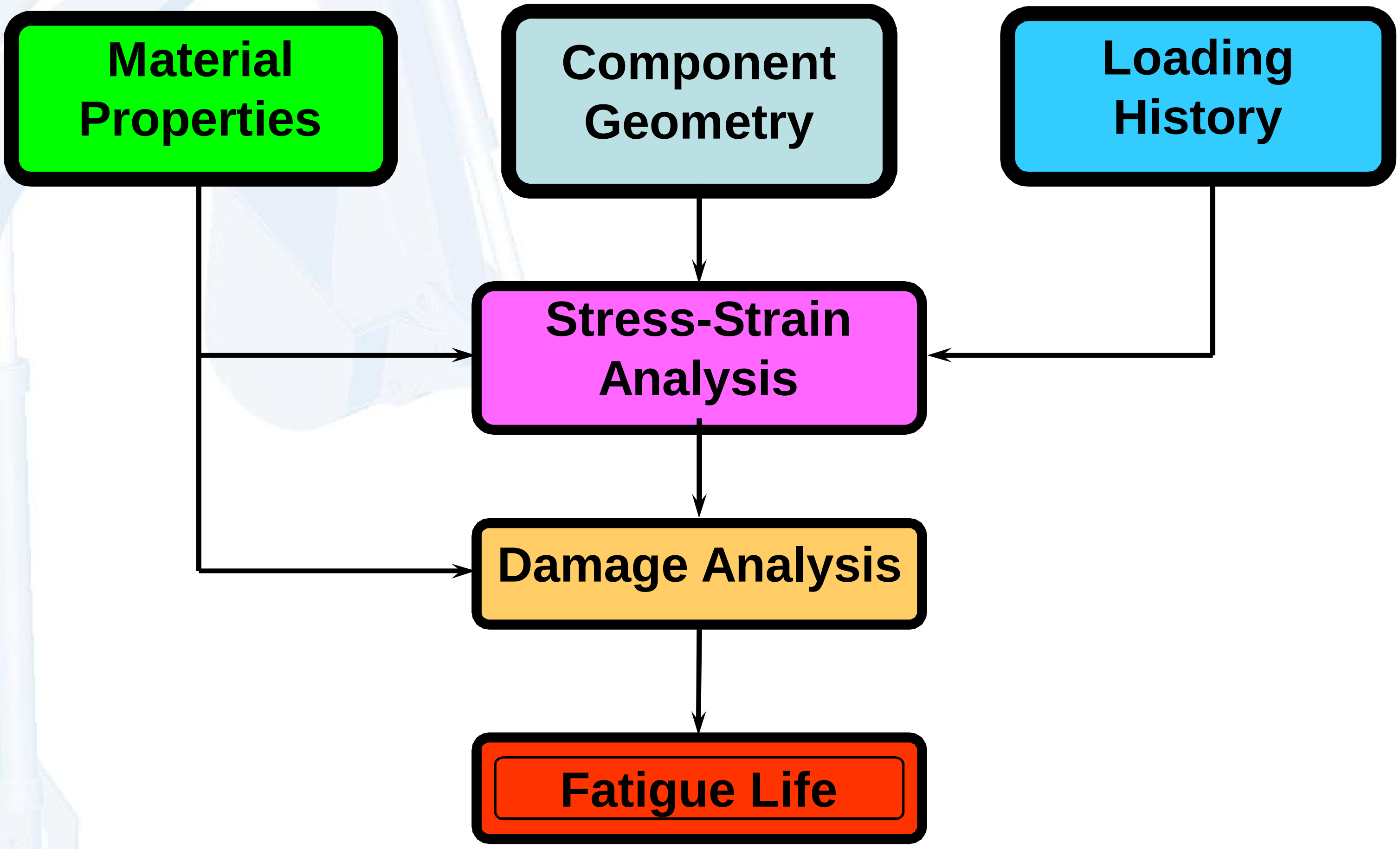
Today

DAY 1

Contemporary Fatigue Analysis Methods

(basics concepts and assumptions)

Information Path for Strength and Fatigue Life Analysis



Stress Parameters Used in Fatigue Analyses

S_n – net nominal stress; S – gross nominal stress

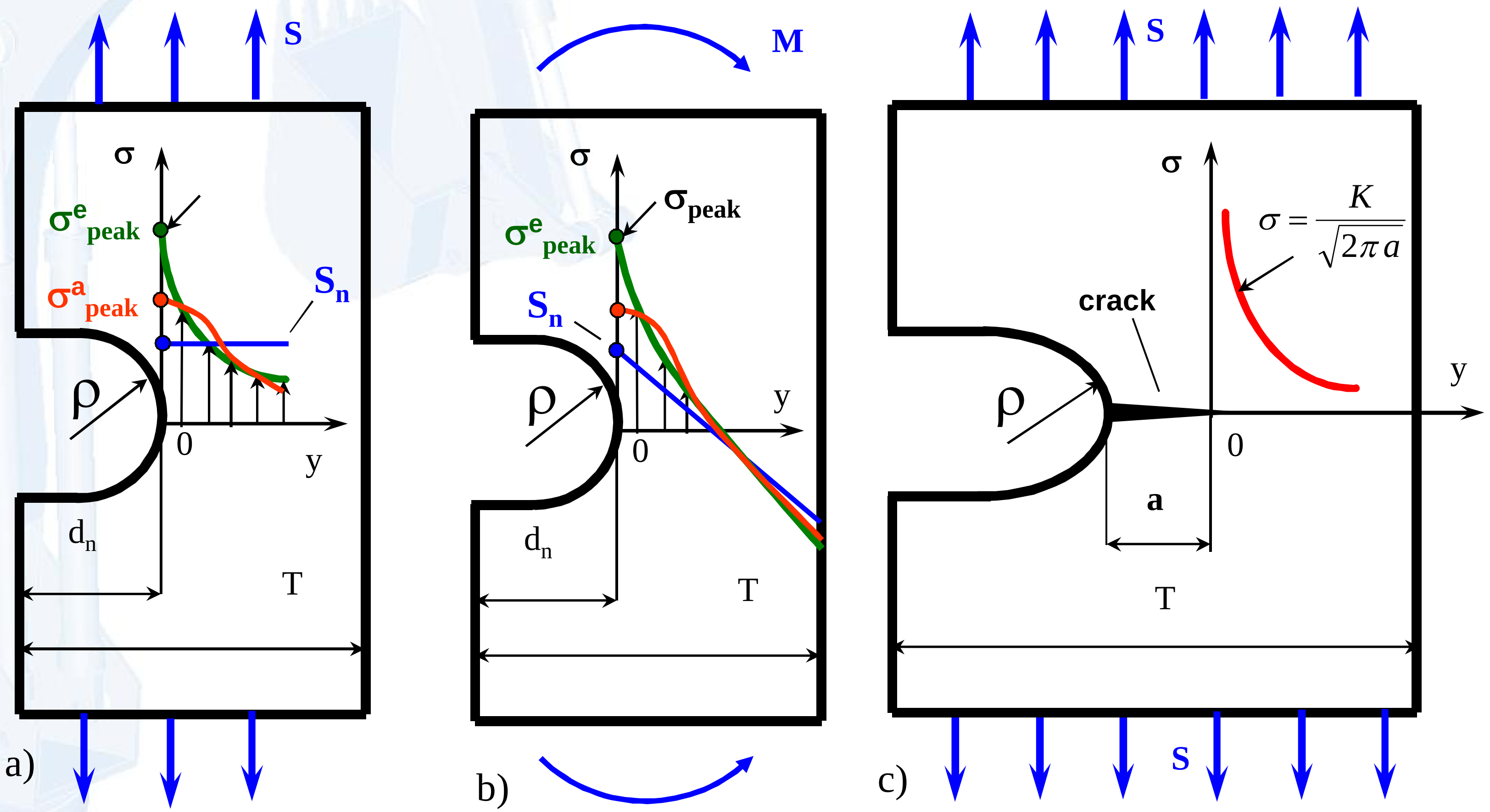
σ^e_{peak} – local linear-elastic notch-tip stress

σ^a_{peak} – local actual elastic-plastic notch-tip stress

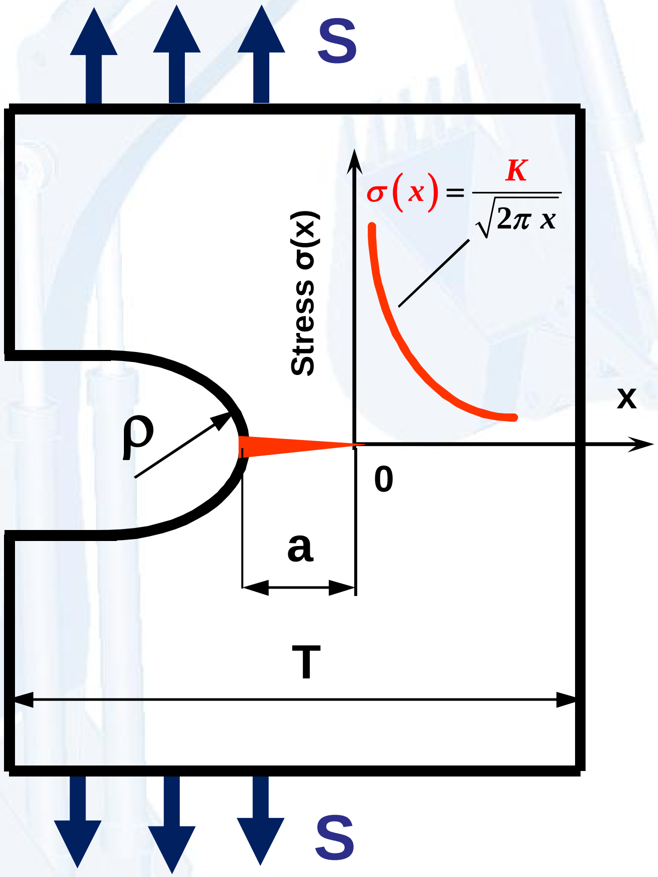
$K_t = \sigma^e_{peak} / S_n$ – stress concentration factor

K – stress intensity factor

$$K = S\sqrt{\pi a} \cdot Y$$



What stress parameter is needed for the Fracture Mechanics based (da/dN-ΔK) fatigue analysis?



The Stress Intensity Factor K characterizing the stress field in the crack tip region is needed!

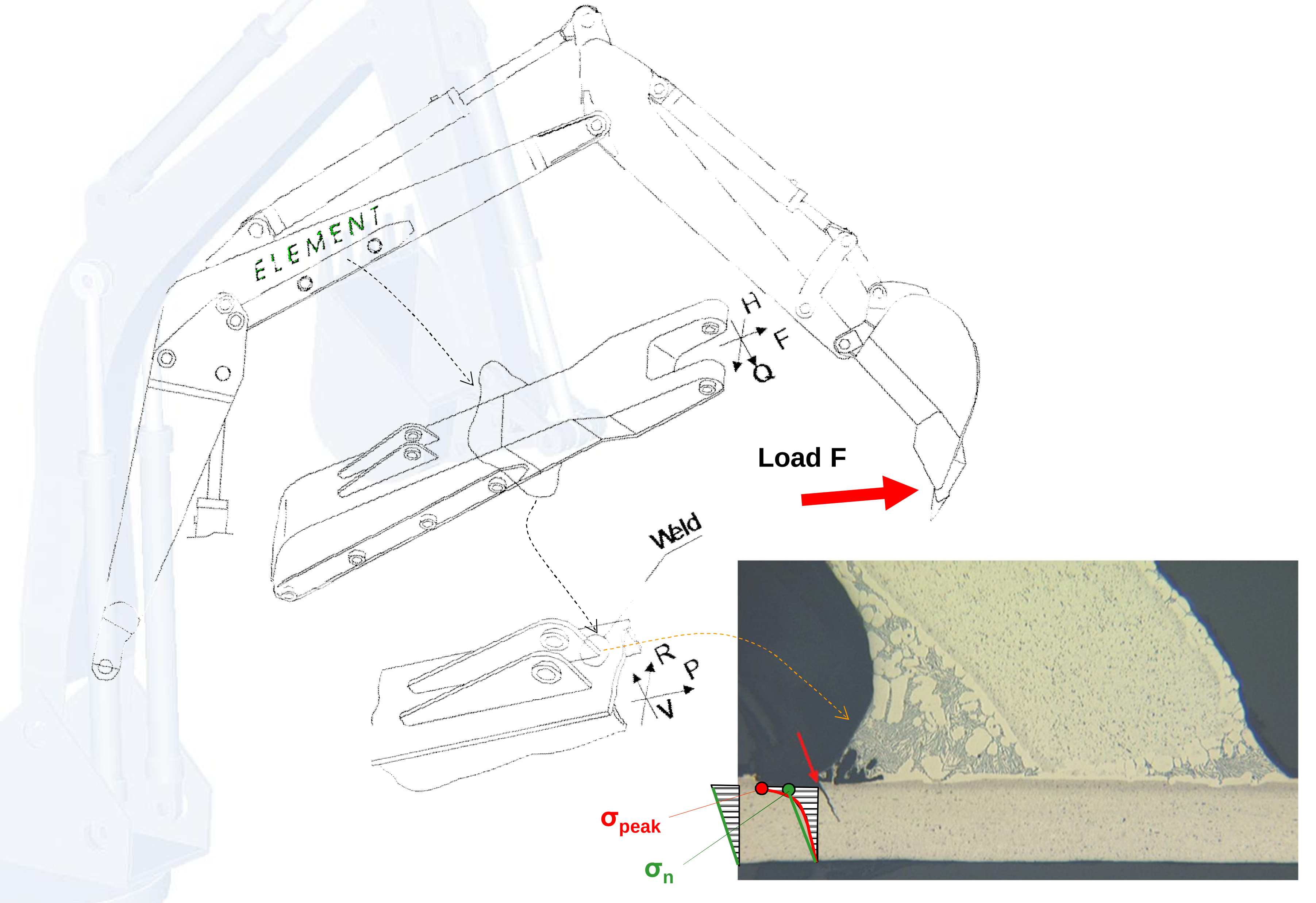
The K factor can be obtained from:

- ready made Handbook solutions *(easy to use but often inadequate in practice)*

- from the near crack tip stress σ(x) distribution or the displacement data obtained from FE analysis of a cracked body *(tedious)*

- from the weight function by using the FE stress analysis data of un-cracked body *(versatile and suitable for FCG analysis)*

Loads and stresses in a structure



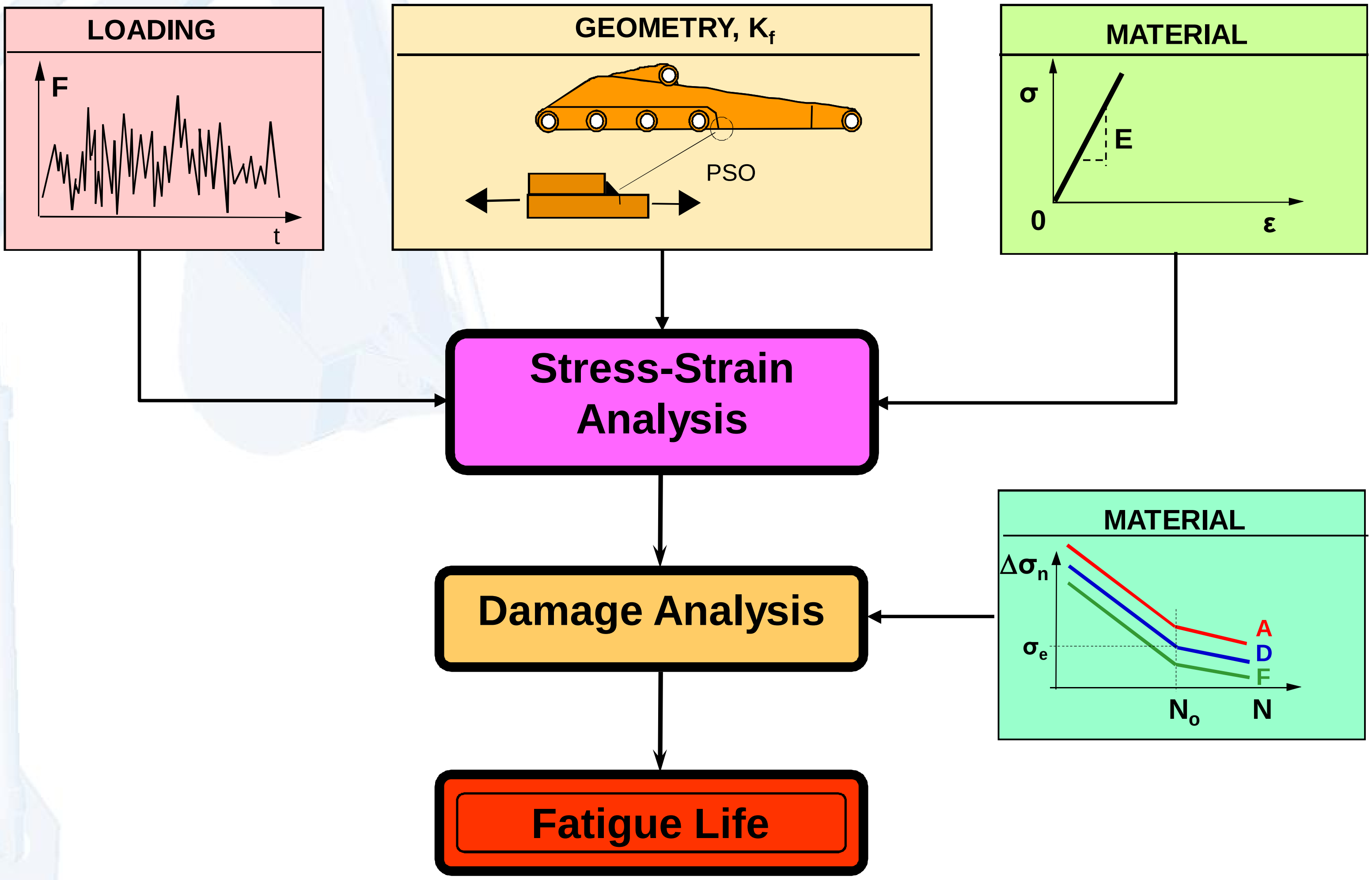
The Most Popular Methods for Fatigue Life Analysis - *outlines*

- **Stress-Life Method** or the **S - N** approach;
  uses the nominal or simple engineering stress ' S ' to quantify fatigue damage

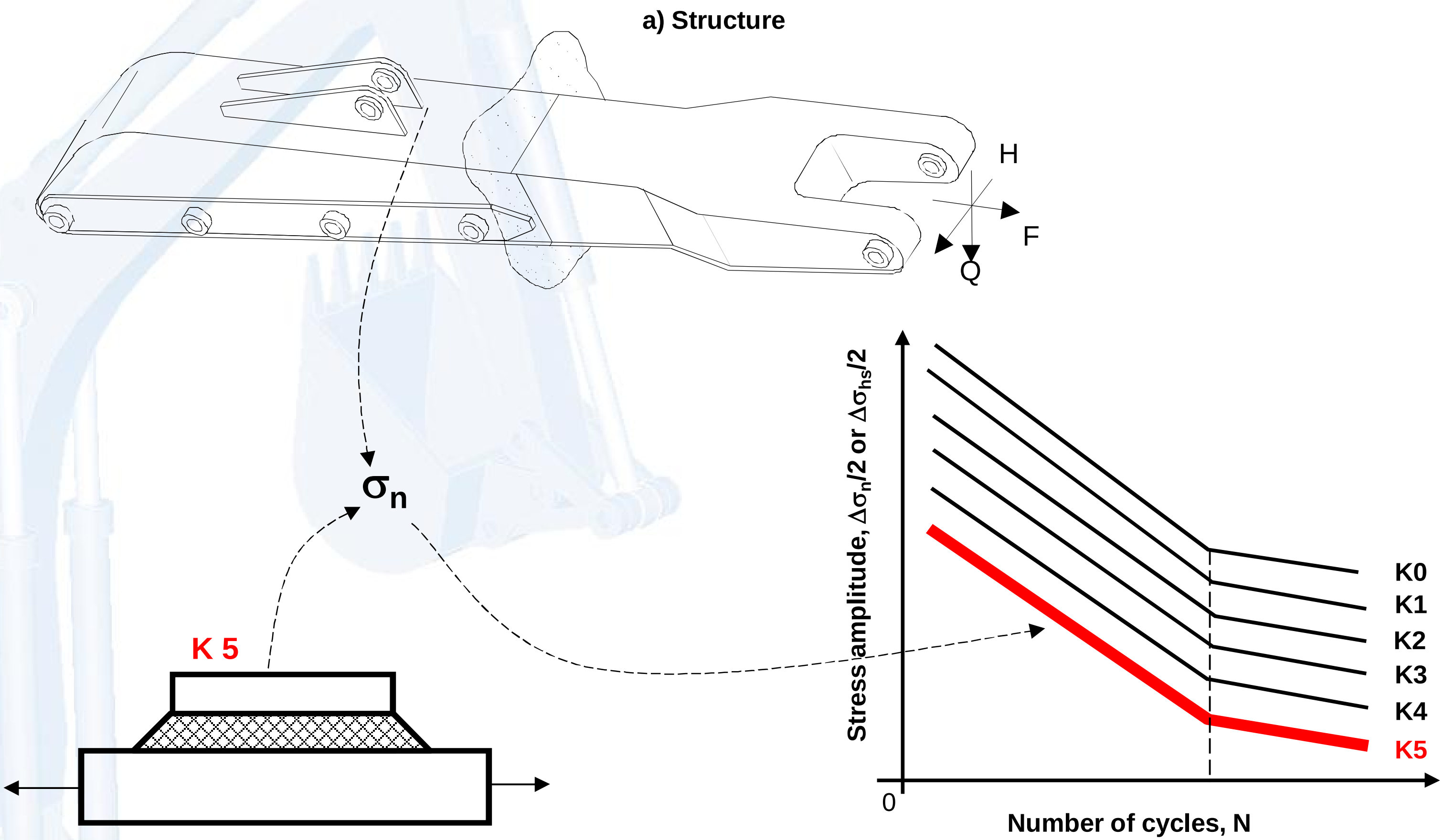
- **Strain-Life Method** or the **ε - N** approach;
  uses the local notch tip strains and stresses to quantify the fatigue damage

- **Fracture Mechanics** or the **da/dN - ΔK** approach;
  uses the stress intensity factor to quatify the fatigue crack growth rate

Information path for fatigue life estimation based on the S-N method



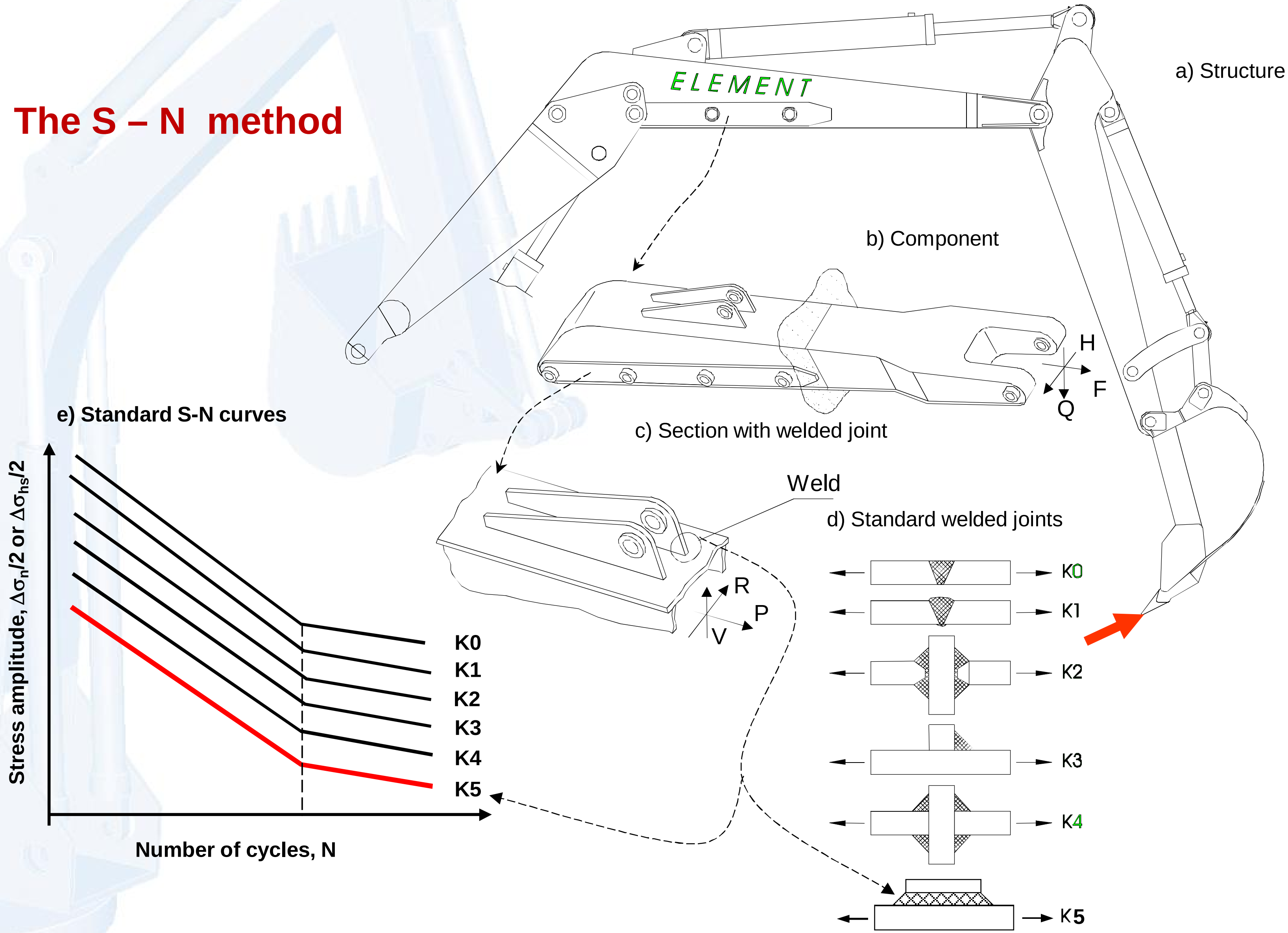
The Similitude Concept in the S-N Method



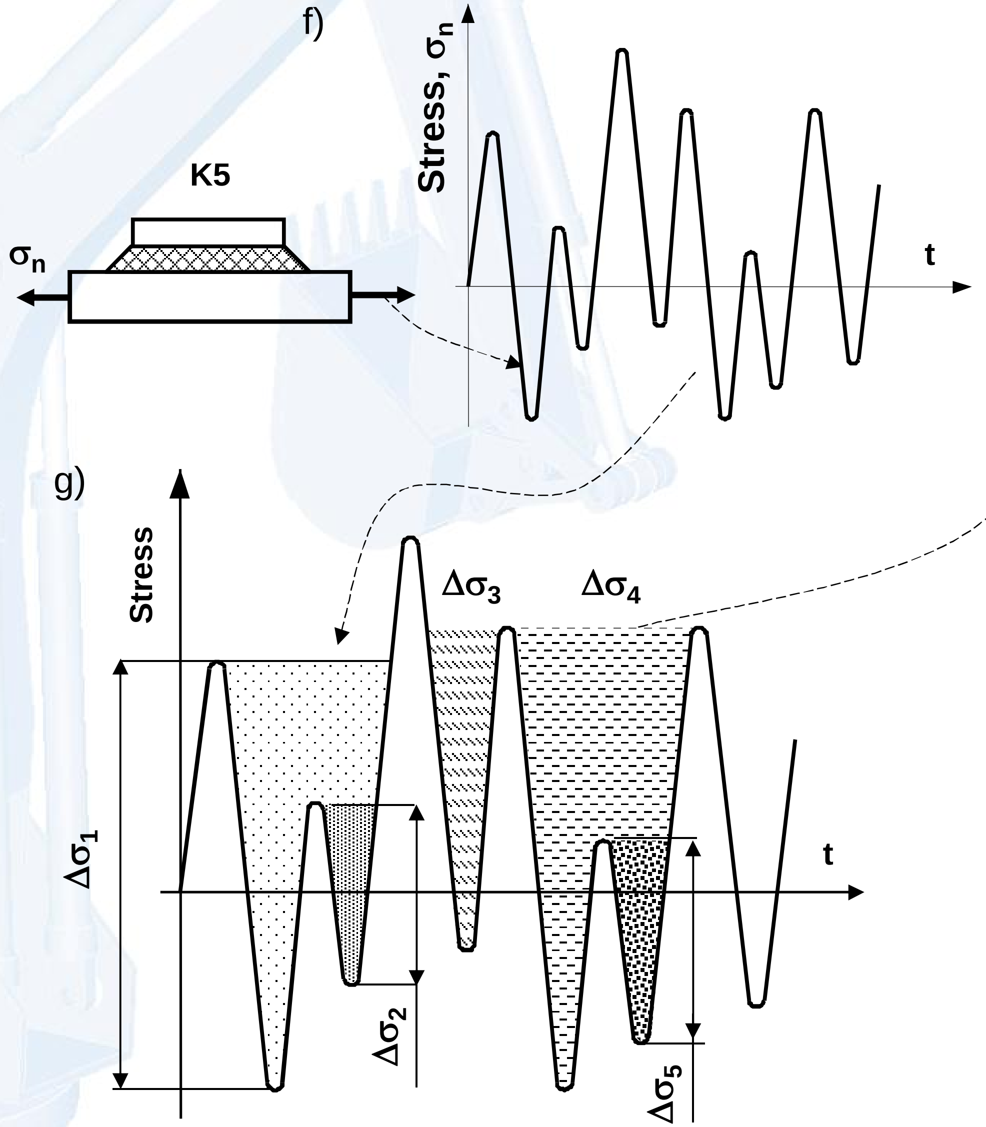
The **Similitude Concept** states that if the nominal stress histories in the structure and in the test specimen are the same, then the fatigue response in each case will also be the same and can be described by the generic S-N curve. It is assumed that such an approach accounts also for the stress concentration, loading sequence effects, manufacturing etc.

Steps in Fatigue Life Prediction Procedure Based on the S-N Approach

The S – N method



Steps in Fatigue Life Prediction Procedure Based on the S-N Approach *(continued)*



h) Fatigue damage:

$$D_1 = \frac{1}{N_1} = \frac{(\Delta\sigma_1)^{m_5}}{C_5}$$

$$D_2 = \frac{1}{N_2} = \frac{(\Delta\sigma_2)^{m_5}}{C_5}$$

$$D_3 = \frac{1}{N_3} = \frac{(\Delta\sigma_3)^{m_5}}{C_5}$$

$$D_4 = \frac{1}{N_4} = \frac{(\Delta\sigma_4)^{m_5}}{C_5}$$

$$D_5 = \frac{1}{N_5} = \frac{(\Delta\sigma_5)^{m_5}}{C_5}$$

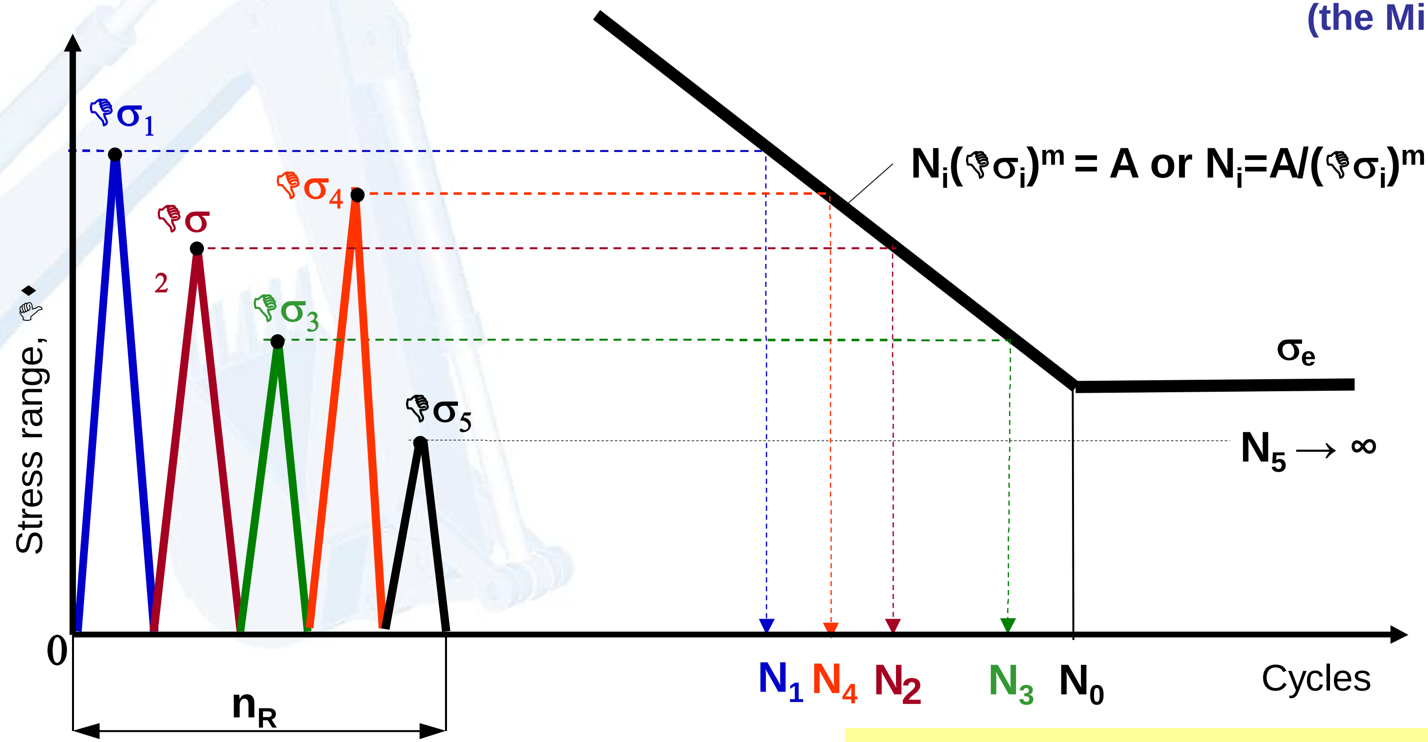
i) Total damage:

$$D = D_1 + D_2 + D_3 + D_4 + D_5;$$

j) **Fatigue life: N_{blck}=1/D**

The linear hypothesis of Fatigue Damage Accumulation

(the Miner rule)



$$D_1 = \frac{1}{N_1} = \frac{(\Delta\sigma_1)^m}{A}; \quad D_4 = \frac{1}{N_4} = \frac{(\Delta\sigma_4)^m}{A};$$

$$D_2 = \frac{1}{N_2} = \frac{(\Delta\sigma_2)^m}{A}; \quad D_5 = \frac{1}{\infty} = 0; \quad D_3 = \frac{1}{N_3} = \frac{(\Delta\sigma_3)^m}{A};$$

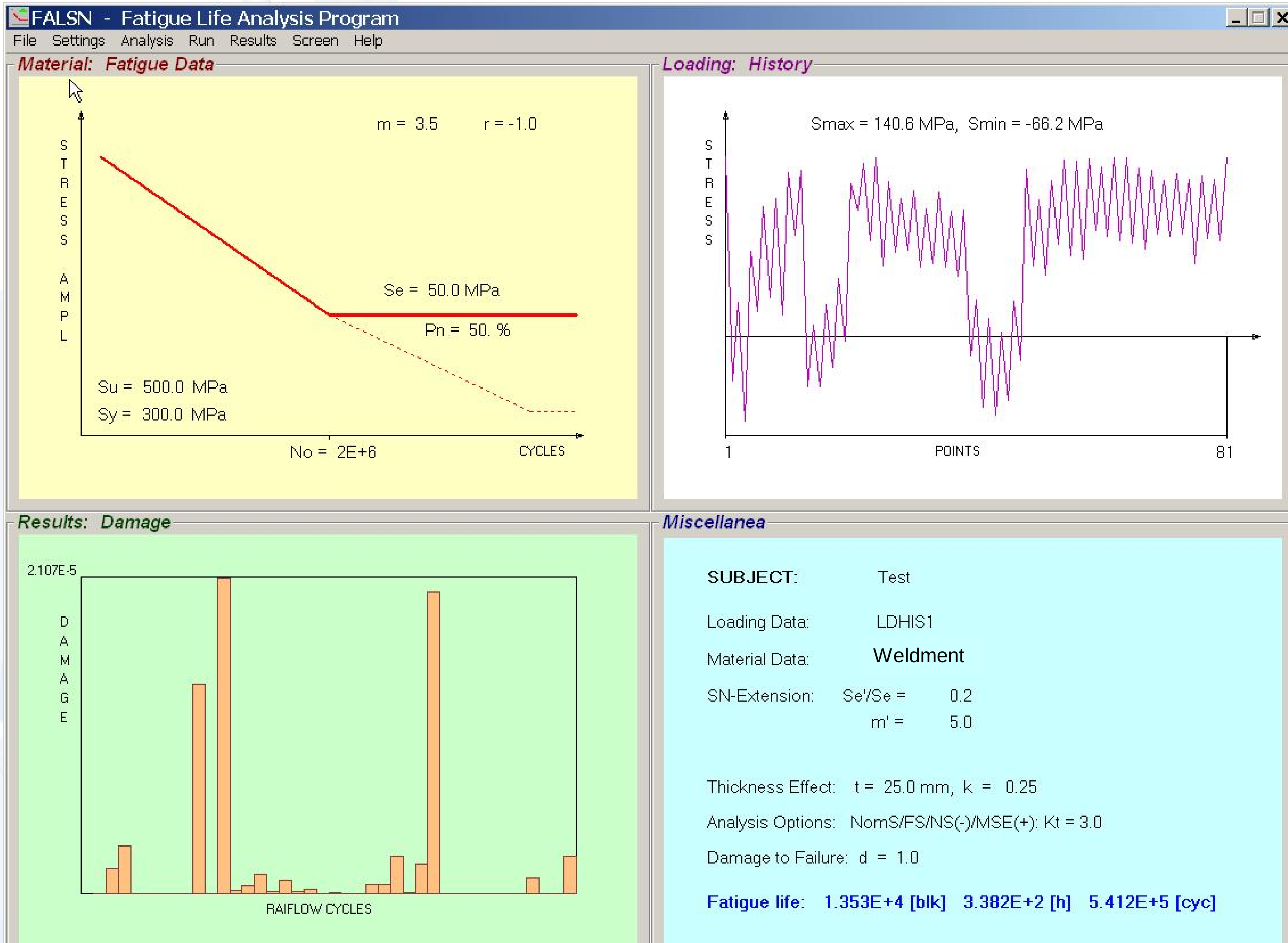
$$D = \sum_{i=1}^{5} D_i = D_1 + D_2 + D_3 + D_4 + D_5$$

$$= \frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3} + \frac{1}{N_4} + \frac{1}{N_5};$$

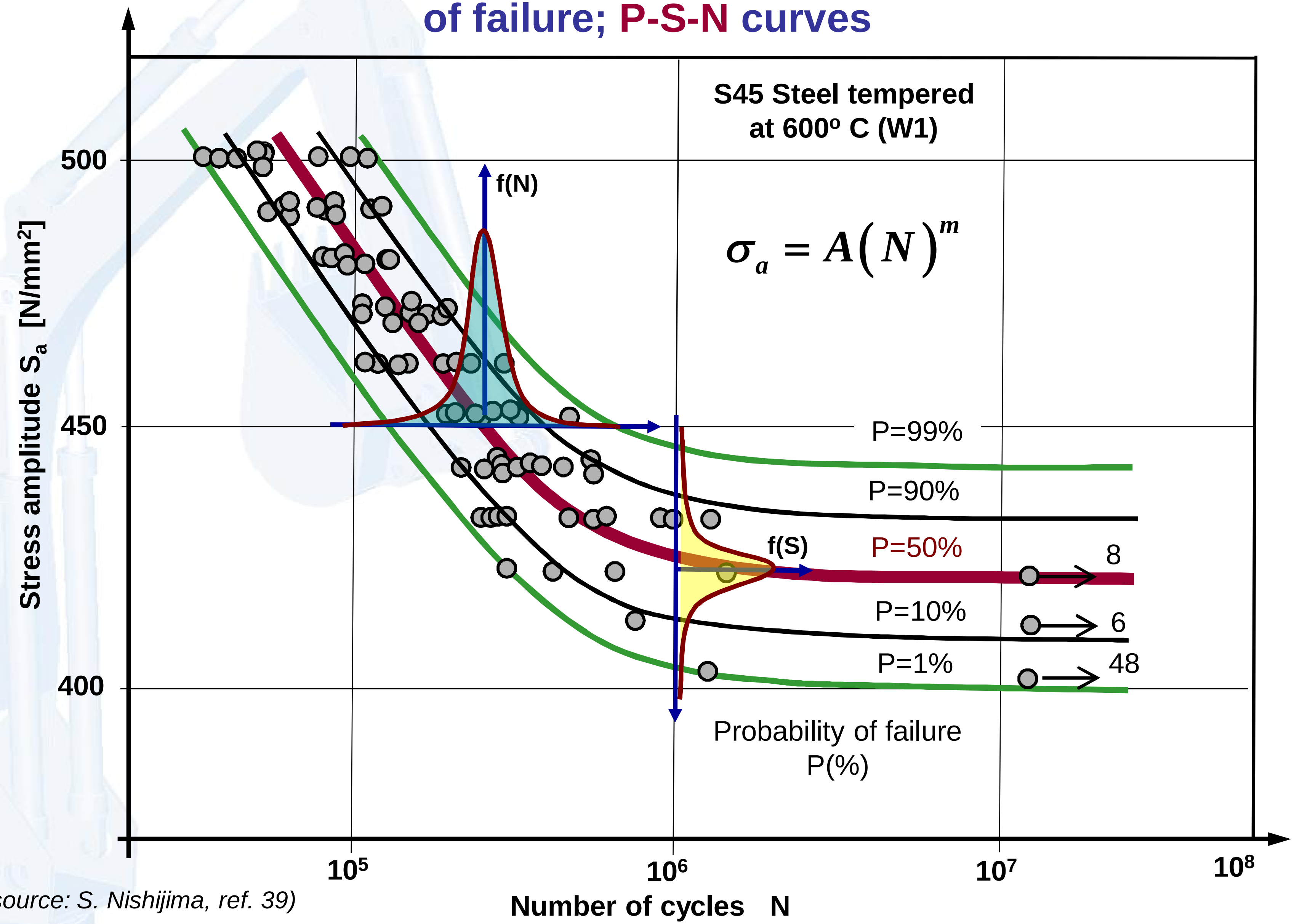
$$if \quad D \geq 1 \rightarrow Failure!!$$

$$L_R = \frac{1}{D} = \frac{1}{1/N_1 + 1/N_2 + 1/N_3 + 1/N_4 + 1/N_5}; \quad N_f = L_R \cdot n_R$$

The FALSN fatigue life estimation software – Typical input and output data

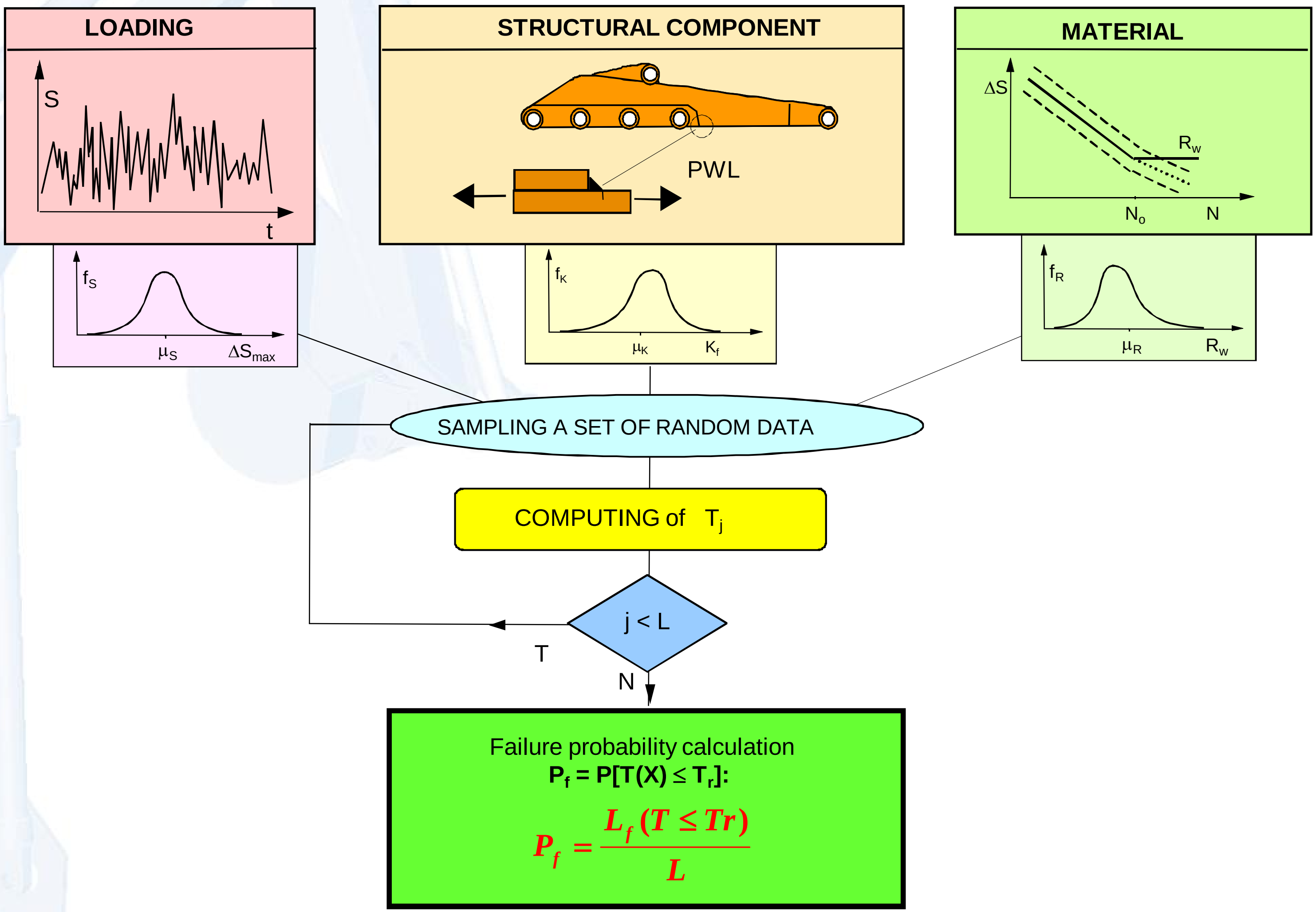


The scatter in fatigue:Fatigue S-N curves for assigned probability of failure; P-S-N curves

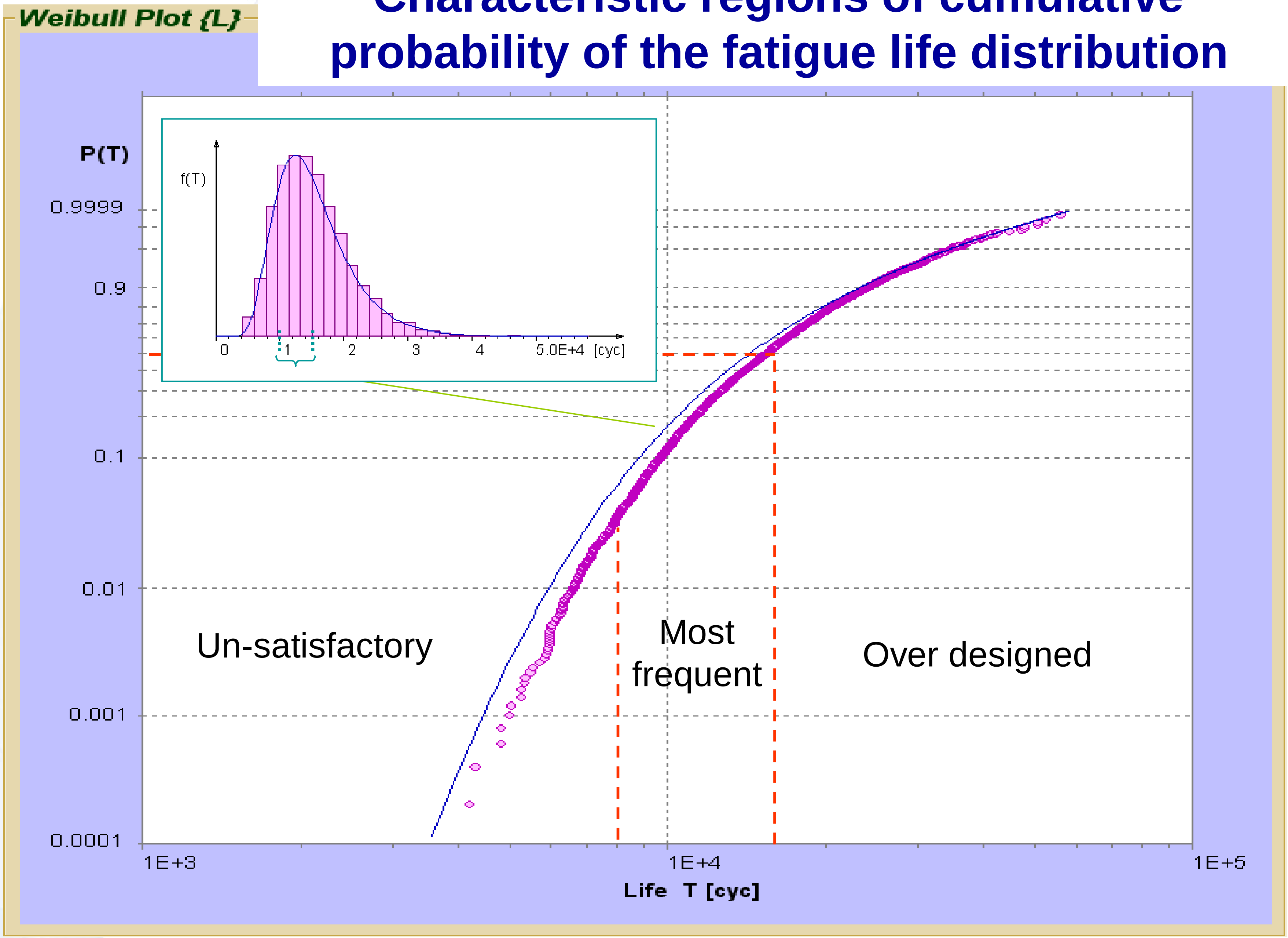


(source: S. Nishijima, ref. 39)

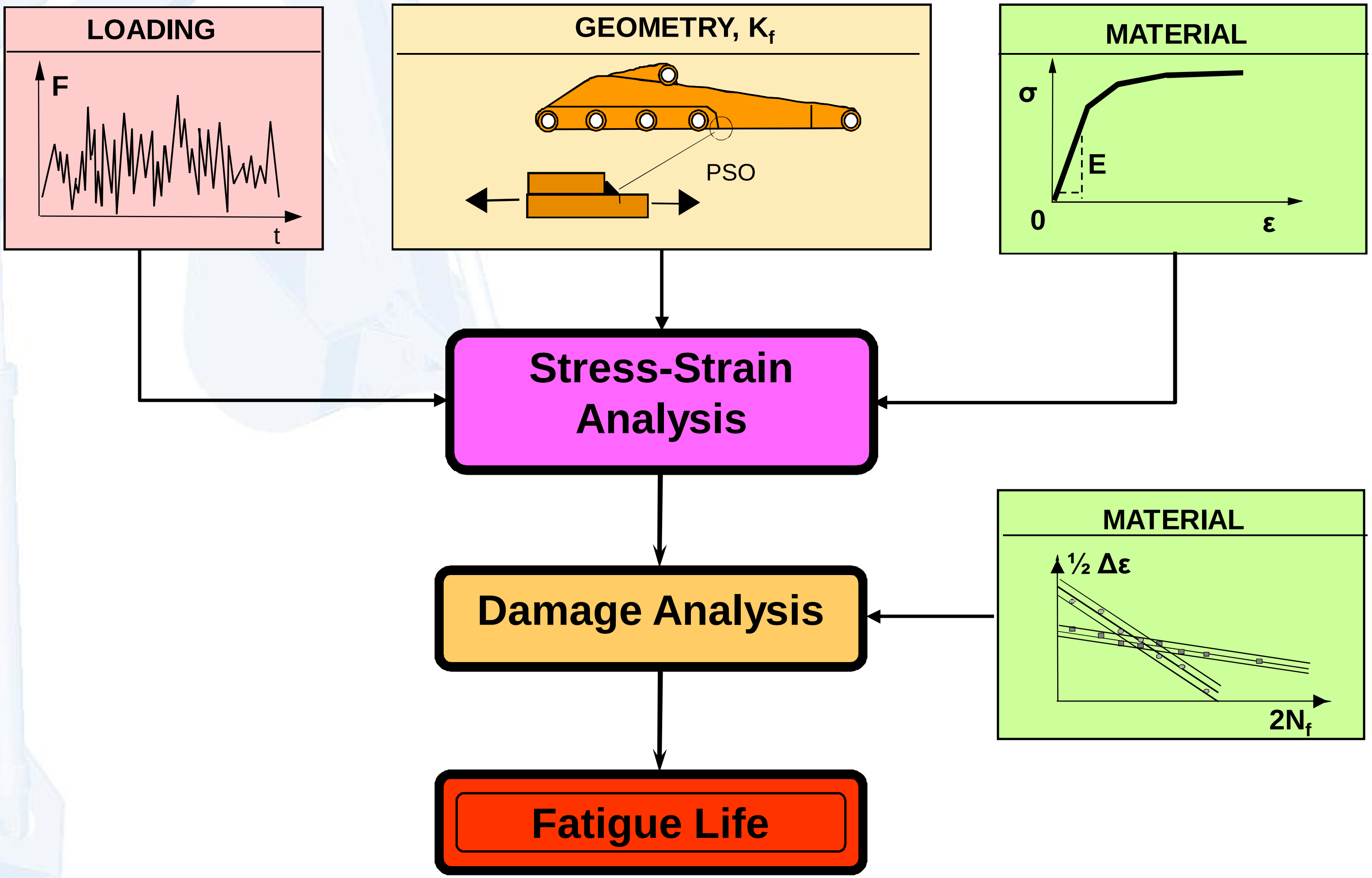
Probabilistic fatigue life assessment



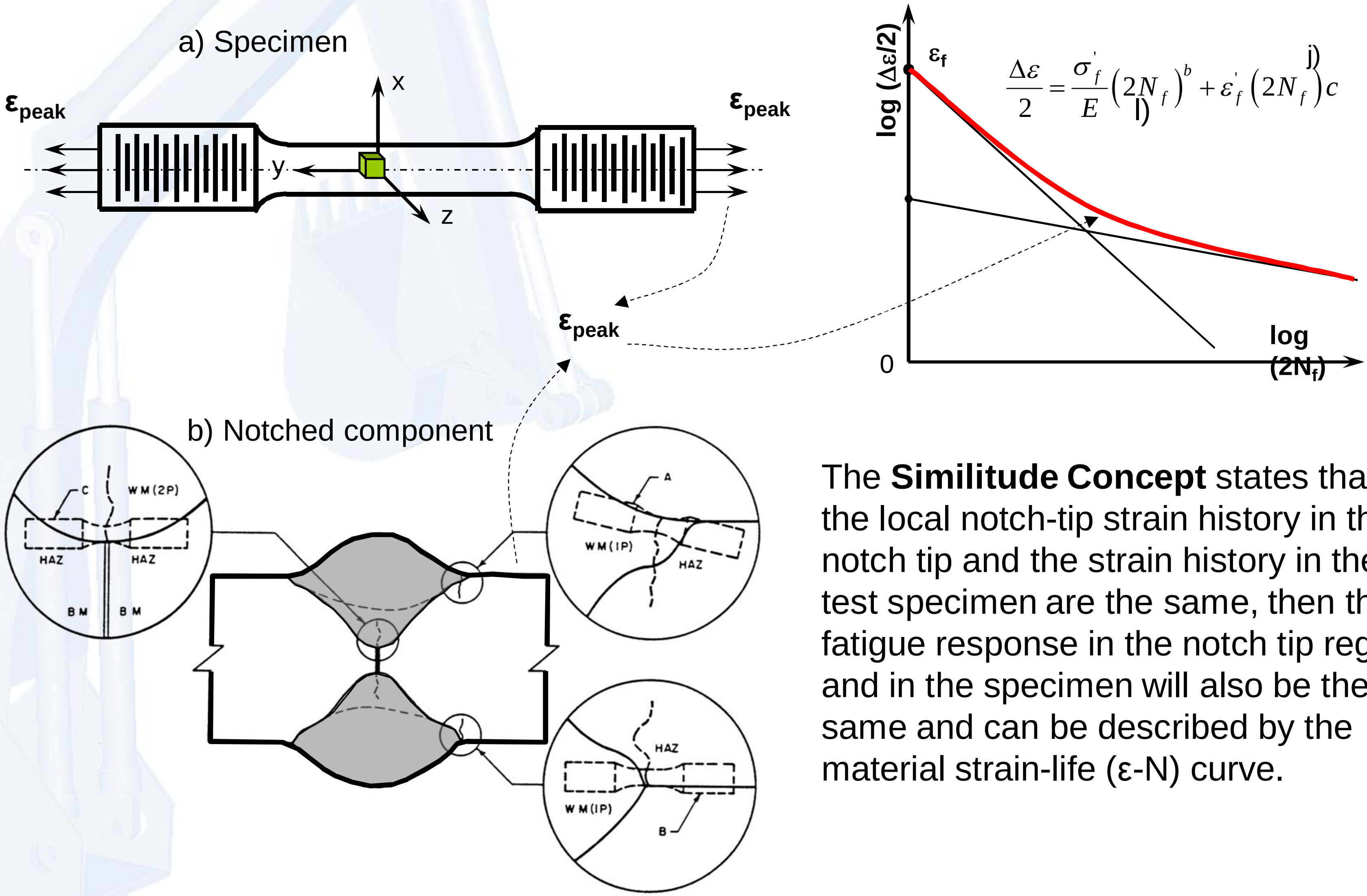
Characteristic regions of cumulative probability of the fatigue life distribution



Information path for fatigue life estimation based on the ε-N method

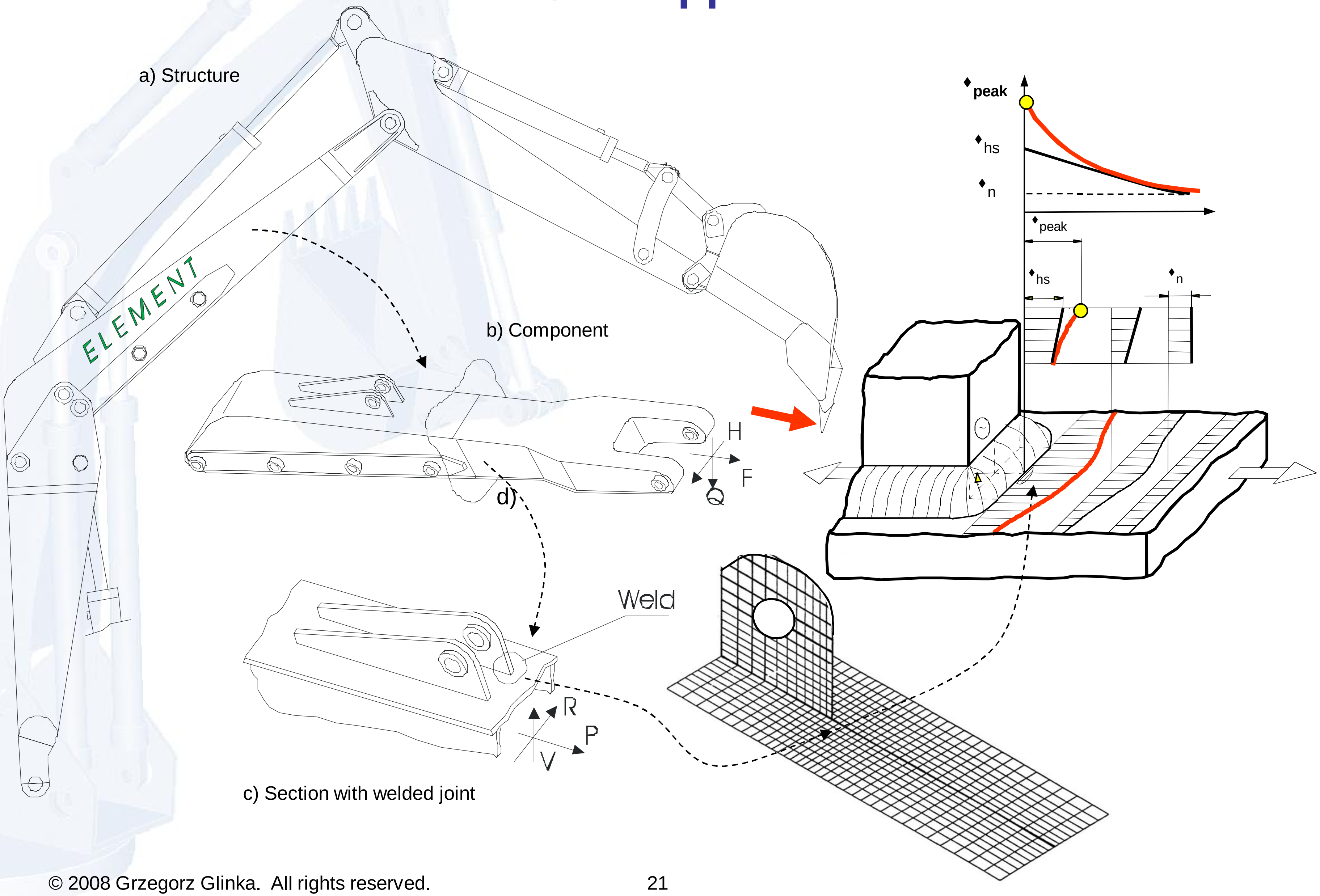


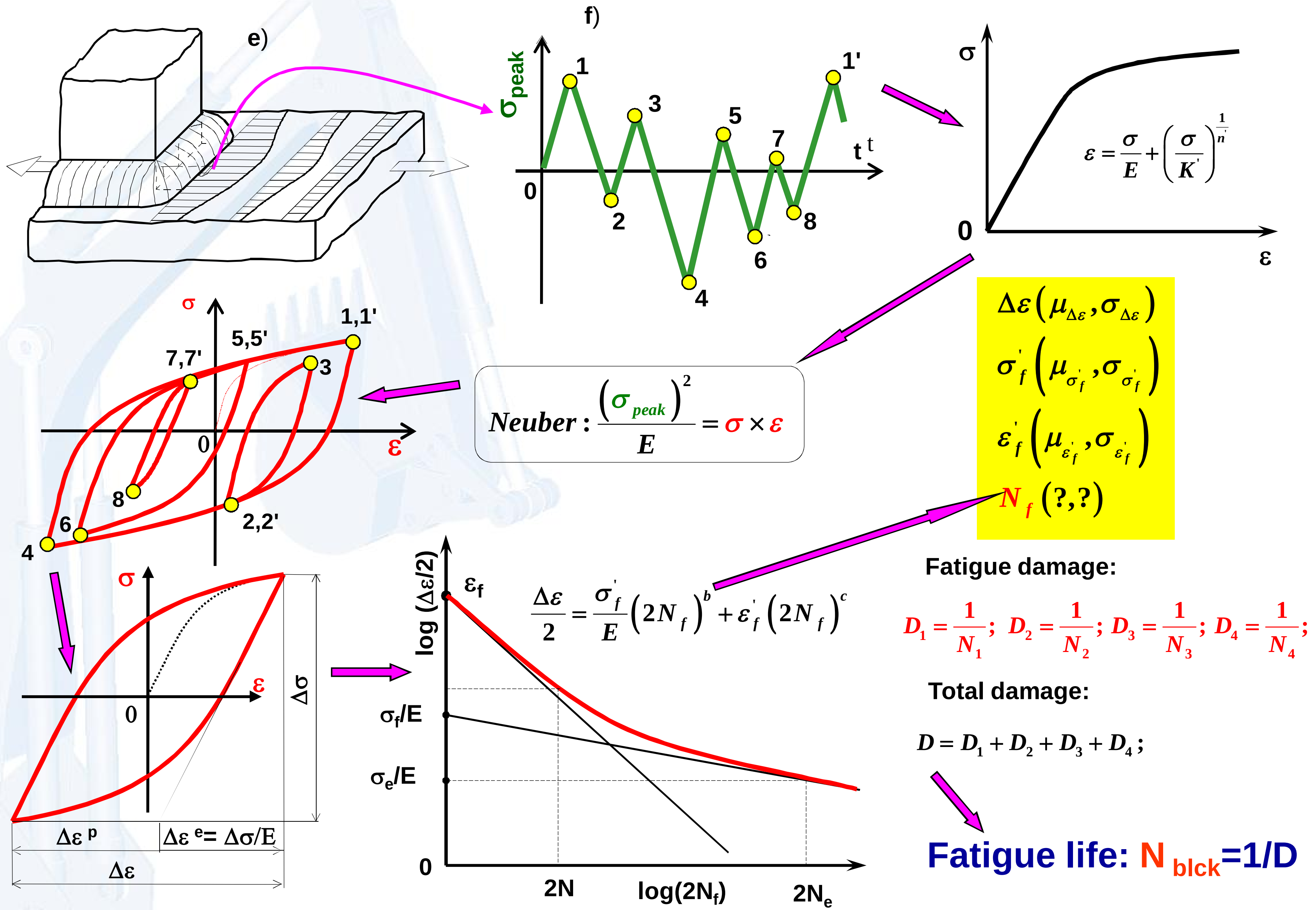
The Similitude Concept in the ε – N Method



The **Similitude Concept** states that if the local notch-tip strain history in the notch tip and the strain history in the test specimen are the same, then the fatigue response in the notch tip region and in the specimen will also be the same and can be described by the material strain-life (ε-N) curve.

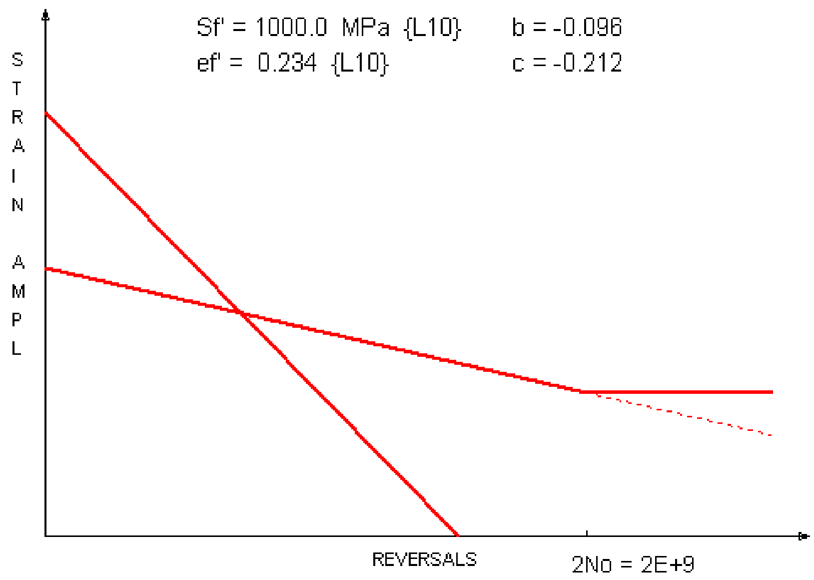
Steps in fatigue life prediction procedure based on the ε - N approach



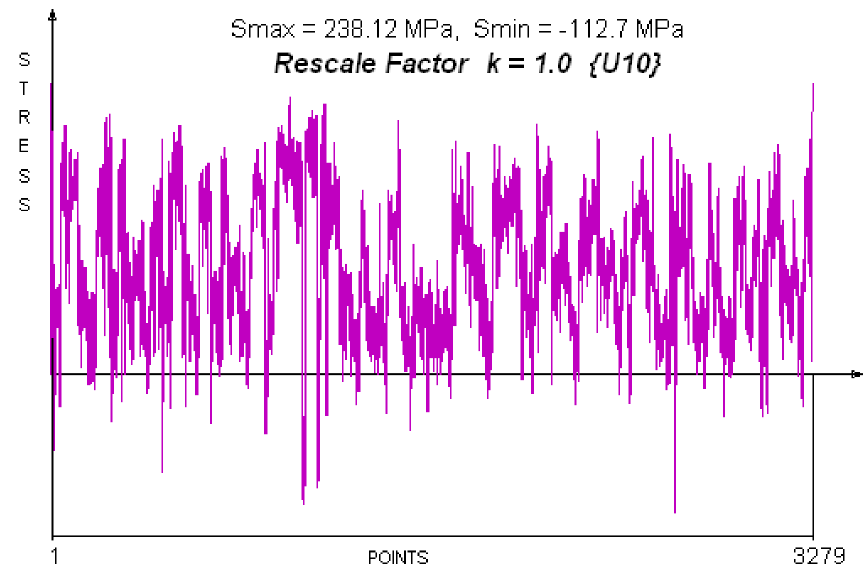


(continued). Steps in fatigue life prediction procedure based on the ε -N approach

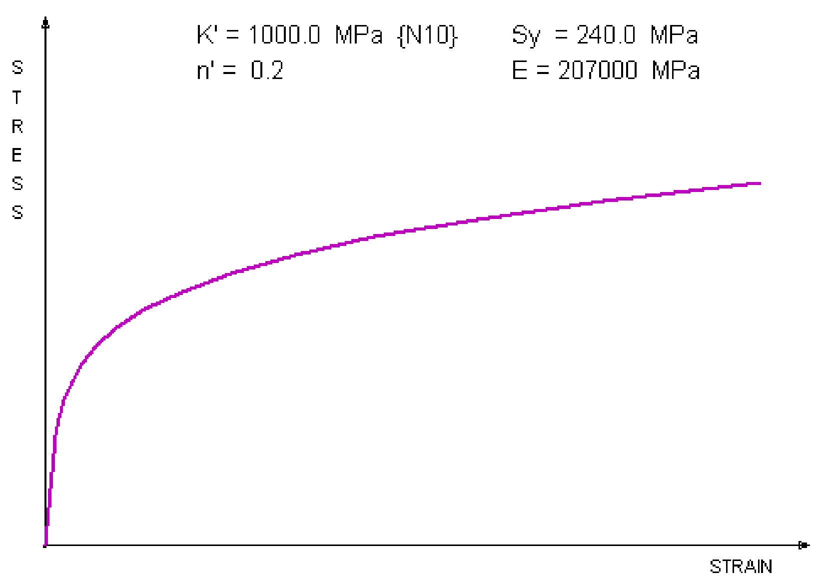
Material: Fatigue Data



Loading: History



Material: Cyclic Curve



Miscellanea

SUBJECT: Show-off

Loading: Batch 1

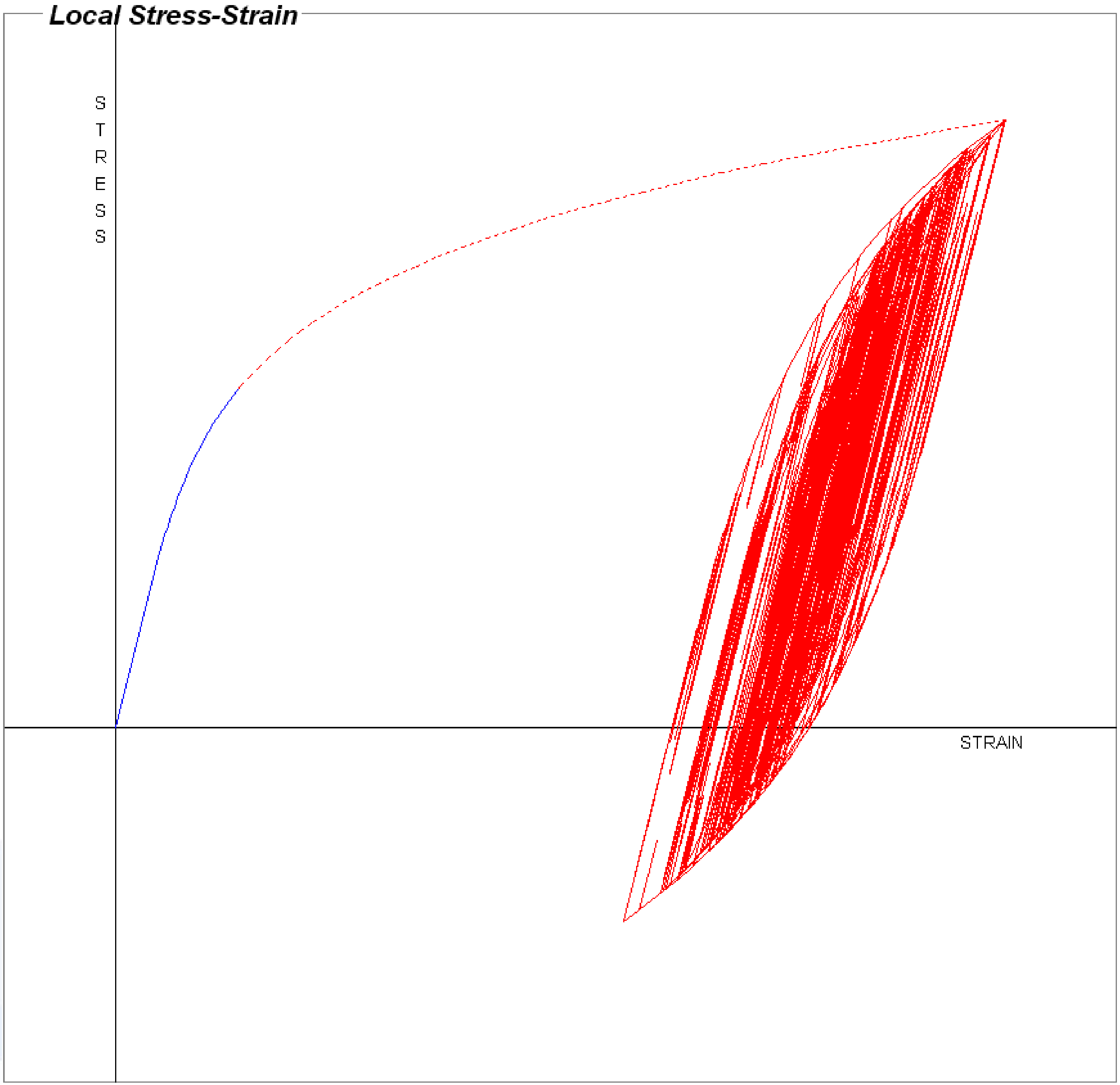
Material: DataBase

Analysis Options: Neuber/PSS/Tens/Morrow

Stress Concentr. Factor: Kt = 3.0 {W15}

Residual Stress: Sr = 220.0 MPa

Reference Life To = 6.8832E+14 [cyc]



Input Data

Loading: History

Ch1 - a3799c06

Smax = 89.41 MPa
Smin = -28.49 MPa
Points: 2673

Material: Cyclic Curve

DataBase
K' = 1000.0 MPa
n' = 0.2

Notch: S-C. Factor

Kt = 3.0

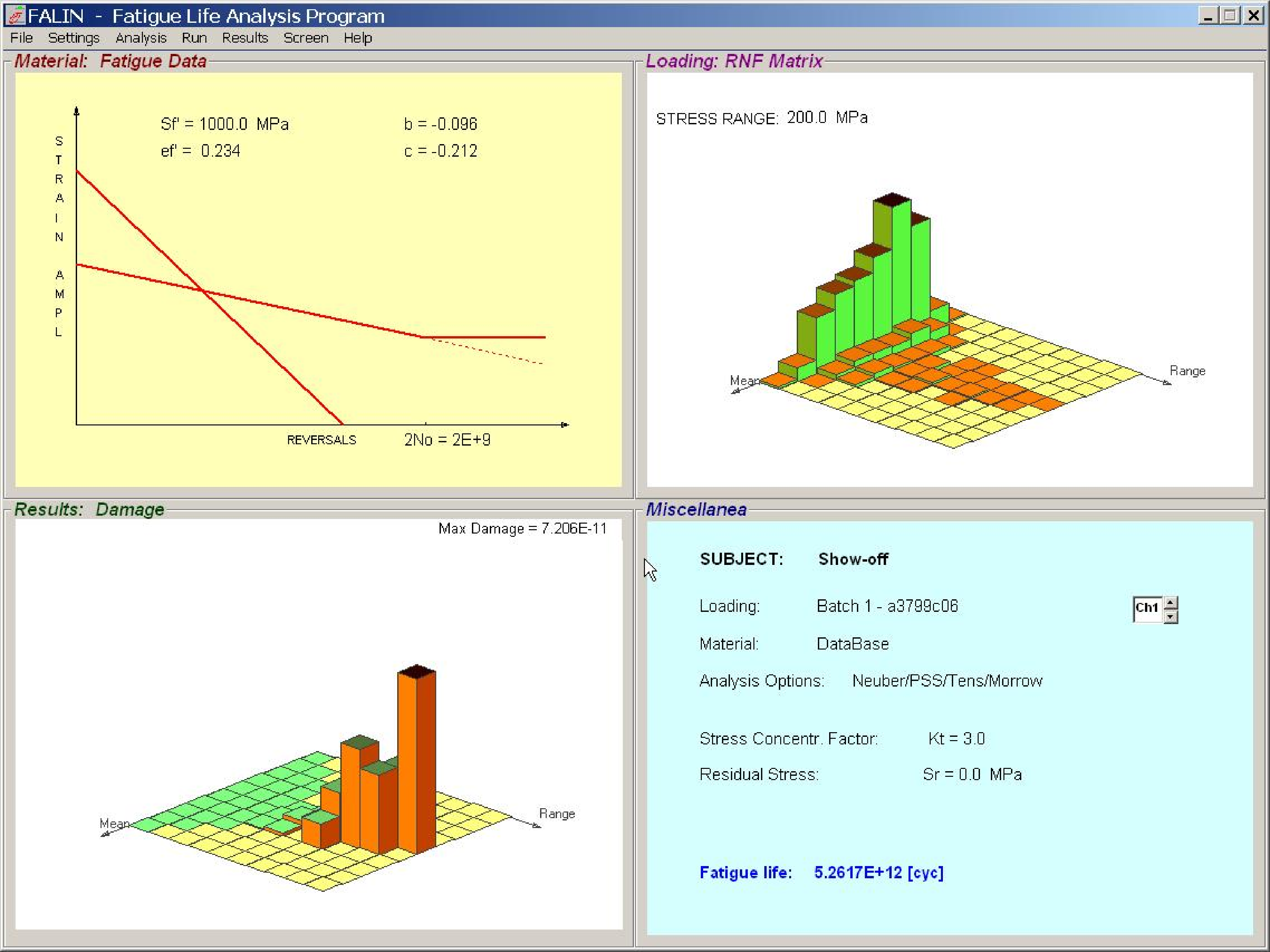
Analysis Options

Neuber/PSS/Tens/Morrow
= = =

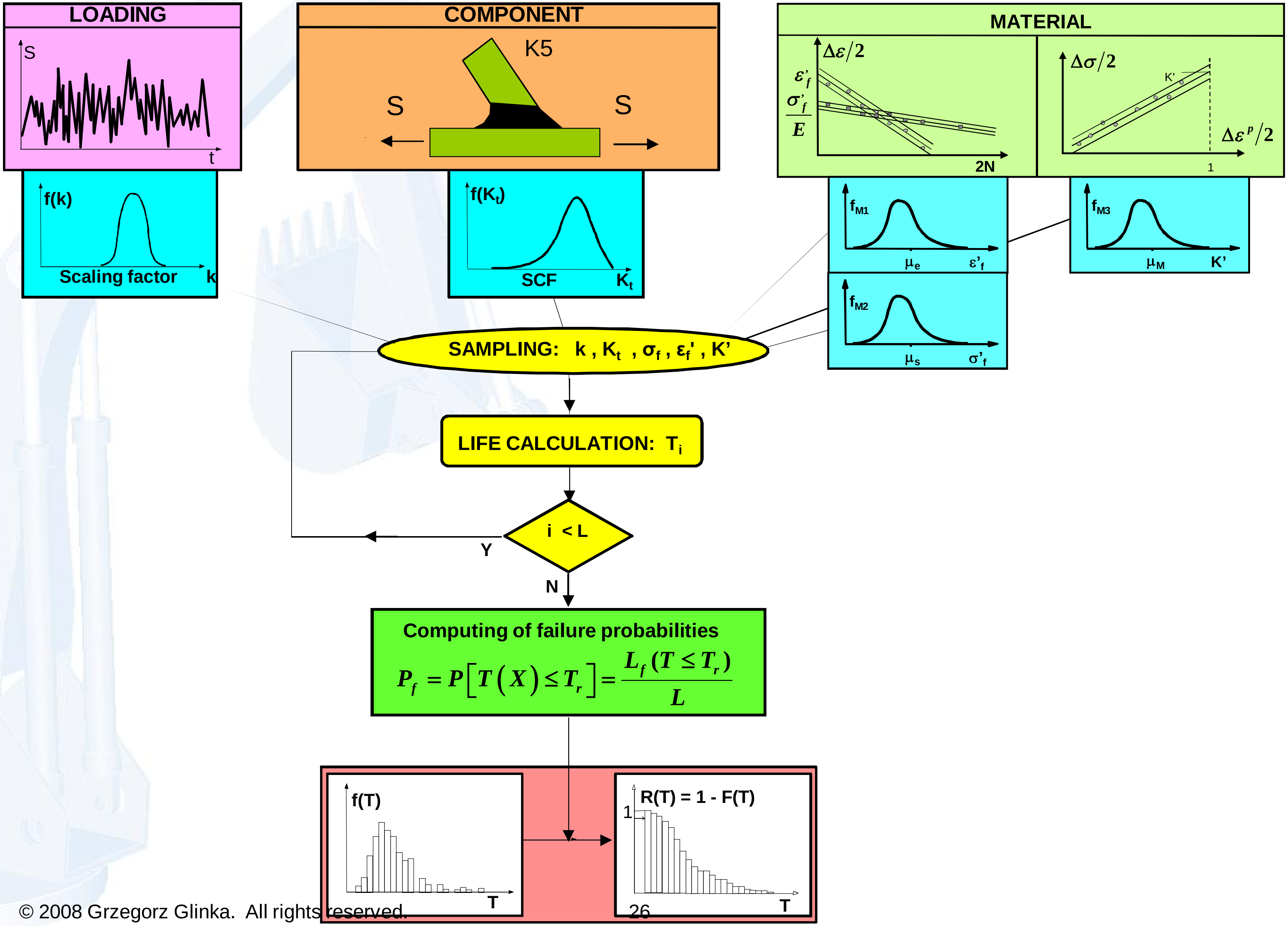
Residual Stress

Sr = 220.0 MPa

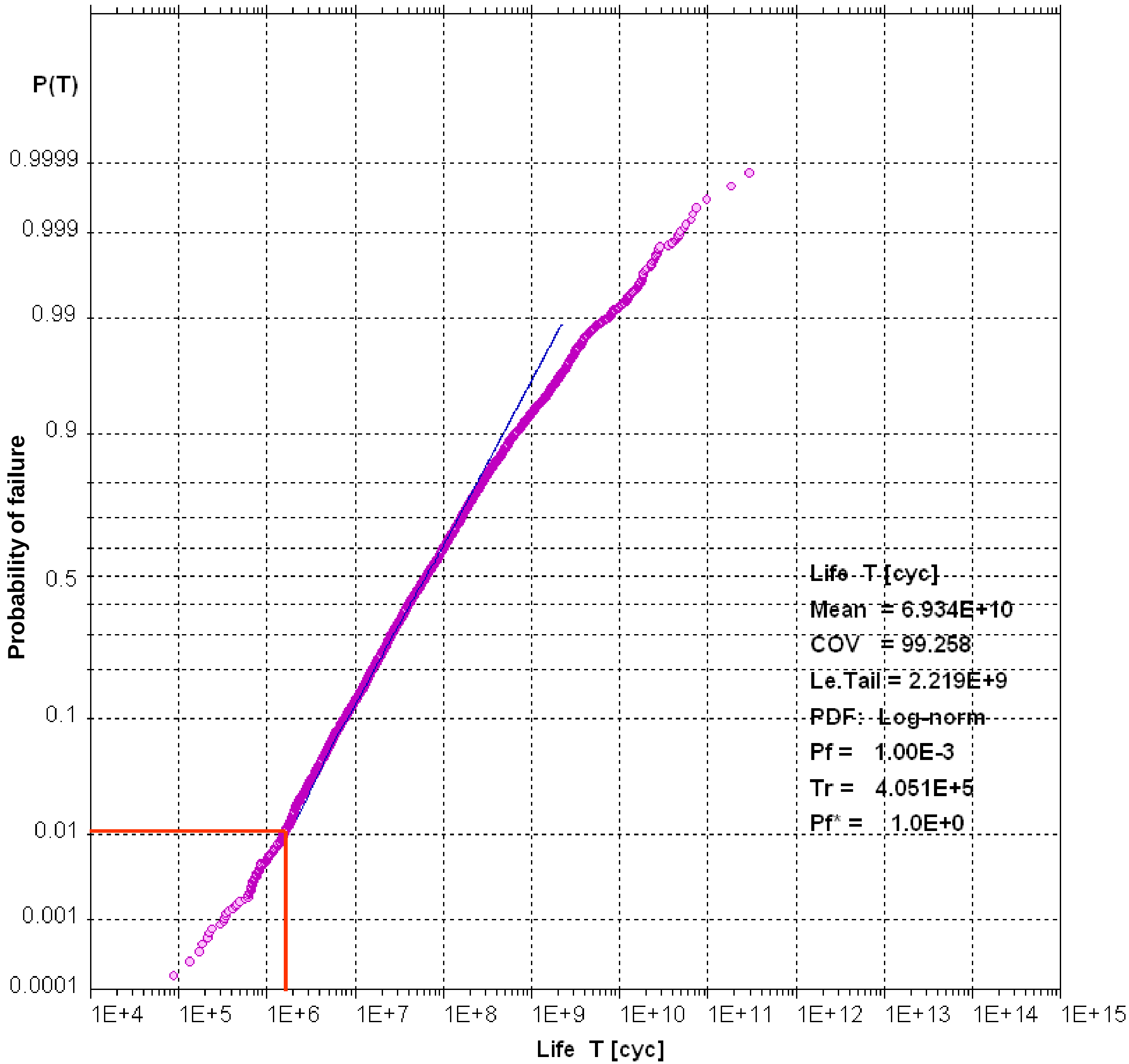
Show-off


FALIN - Fatigue Life Analysis Program
File Settings Analysis Run Results Screen Help
Material: Fatigue Data
Sf' = 1000.0 MPa
b = -0.096
ef' = 0.234
c = -0.212
STRAIN AMPL
REVERSALS
2No = 2E+9
Loading: RNF Matrix
STRESS RANGE: 200.0 MPa
Mean
Range
Results: Damage
Max Damage = 7.206E-11
Mean
Range
Miscellanea
SUBJECT: Show-off
Loading: Batch 1 - a3799c06
Ch1
Material: DataBase
Analysis Options: Neuber/PSS/Tens/Morrow
Stress Concentr. Factor: Kt = 3.0
Residual Stress: Sr = 0.0 MPa
Fatigue life: 5.2617E+12 [cyc]

Three basic sets of input data for the evaluation of the Fatigue Crack Initiation Life and Reliability (the ε – N approach)



Probability Plot {Log-norm}



Input Data

Loading: History

Ch2 - a3799f06
Smax = 238.1 MPa
Smin = -112.7 MPa
Points: 3279
Rescale Factor: {U10}

Material Properties

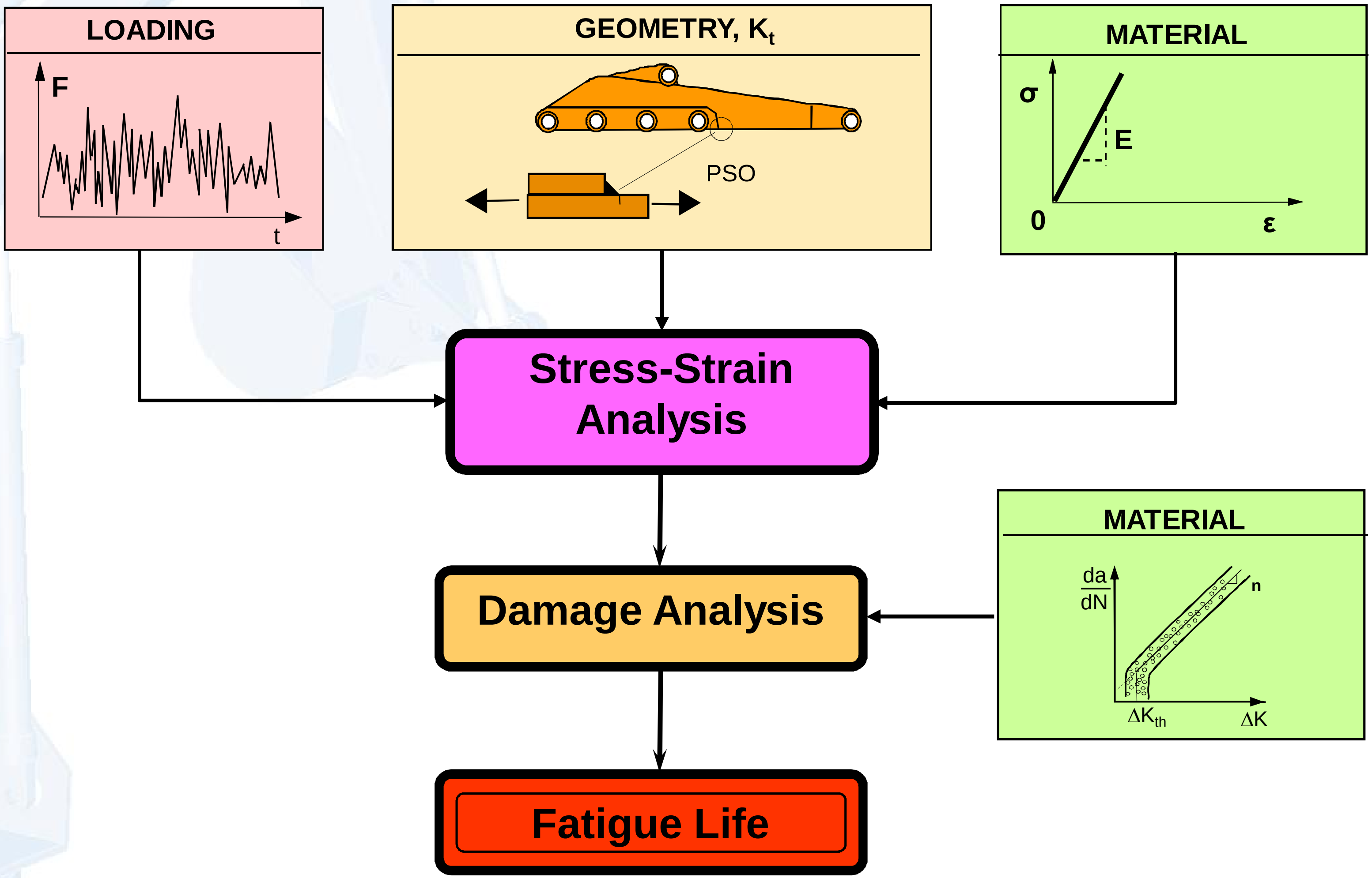
DataBase
Sf' = 1000.0 MPa {L10}
b = -0.096
ef' = 0.234 {L10}
c = -0.212
K' = 1000.0 MPa {N10}
n' = 0.2
Sy = 240.0 MPa
E = 207000 MPa

Analysis Options

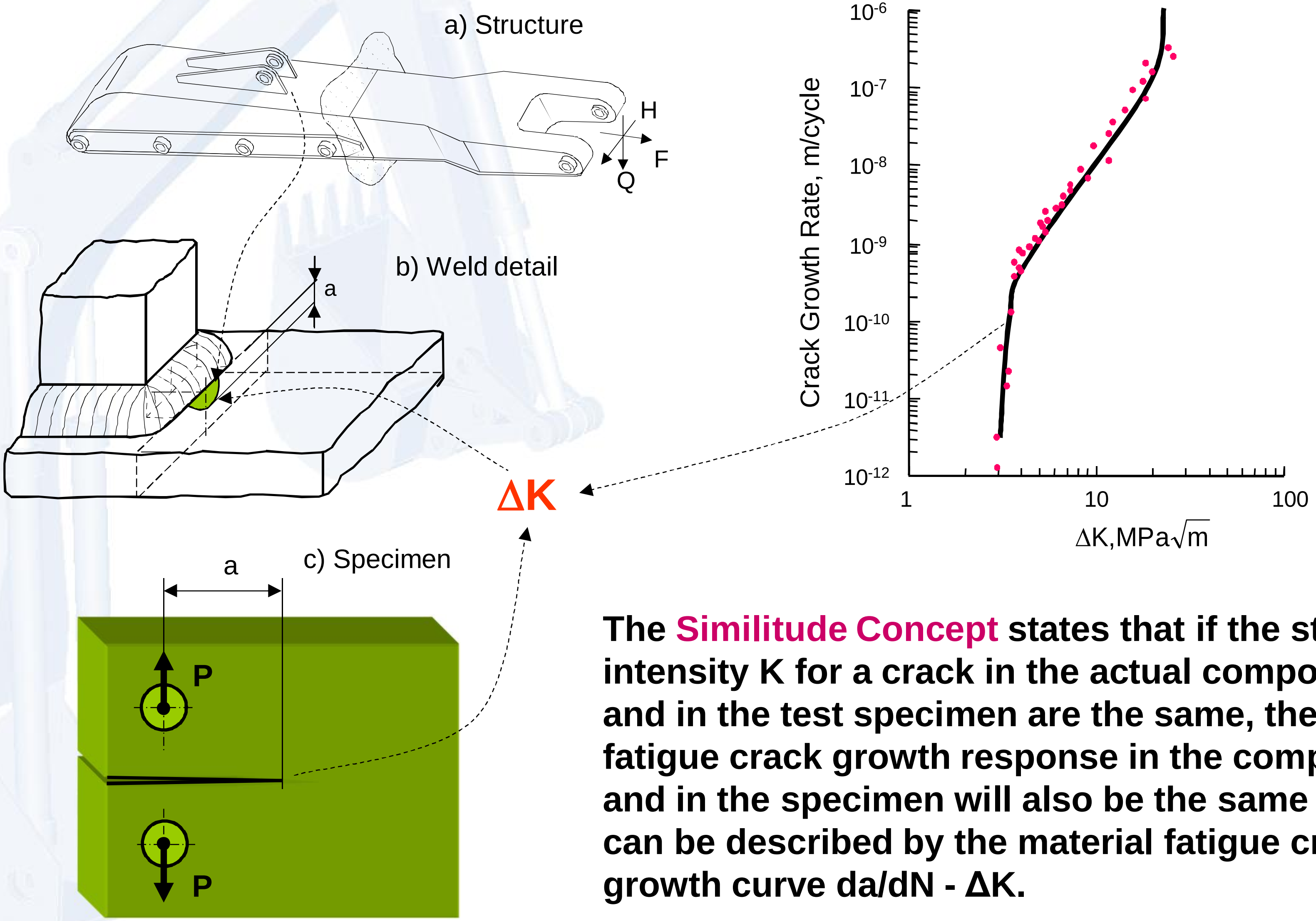
Neuber/PSS/Tens/FSA/FL
= = =
Kt = 3.0 {W15}
Sr = 220.0 MPa {N10}
Sample = 10000

Show-off

Information path for fatigue life estimation based on the da/dN-ΔK method

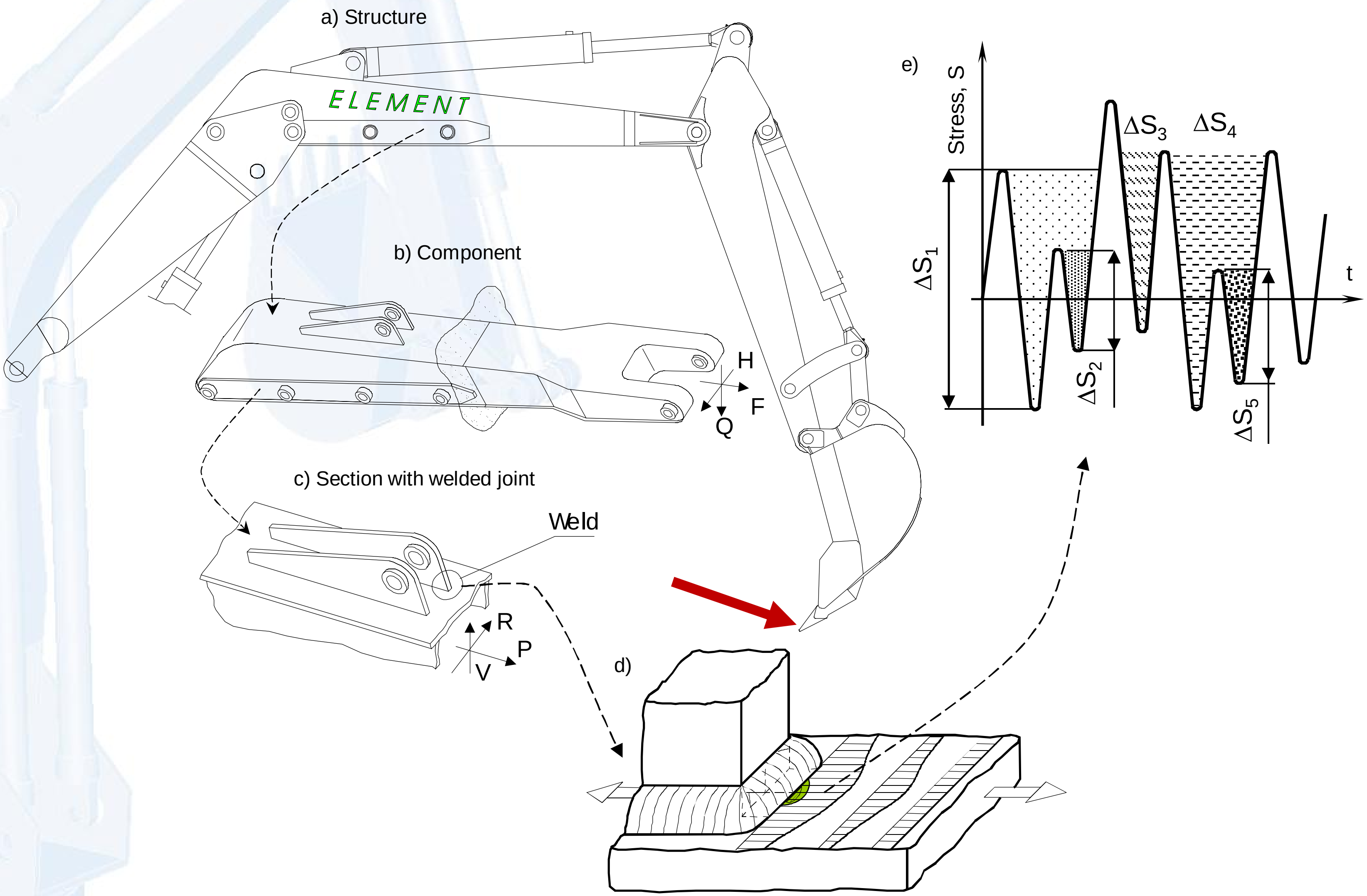


The Similitude Concept in the da/dN – ΔK Method



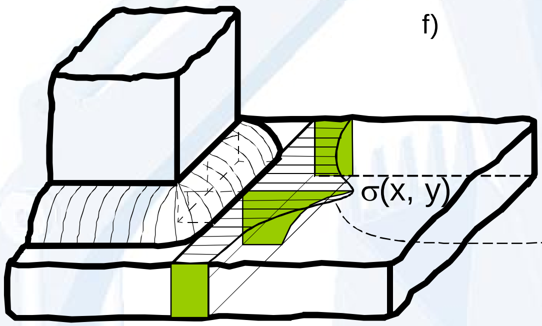
The Similitude Concept states that if the stress intensity K for a crack in the actual component and in the test specimen are the same, then the fatigue crack growth response in the component and in the specimen will also be the same and can be described by the material fatigue crack growth curve da/dN - ΔK.

Steps in the Fatigue Life Prediction Procedure Based on the da/dN-ΔK Approach



Steps in Fatigue Life Prediction Procedure Based on the da/dN-ΔK Approach *(cont'd)*

f)



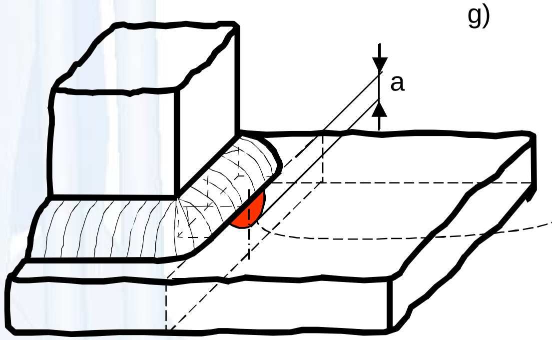
Stress intensity factor, K (indirect method)

Weight function, m(x,y)

$$K = \iint_A \sigma(x,y)\, m(x,y)\, dxdy$$

$$Y = \frac{K}{\sigma_n \sqrt{\pi a}}$$

g)



Stress intensity factor, K (direct method)

$$K_I = \sigma_{yFE}\sqrt{2\pi x_{FE}}$$

or

$$K = \sqrt{E\frac{dU}{da}} = \sqrt{EG}$$

$$Y = \frac{K}{\sigma_n \sqrt{\pi a}}$$

h)

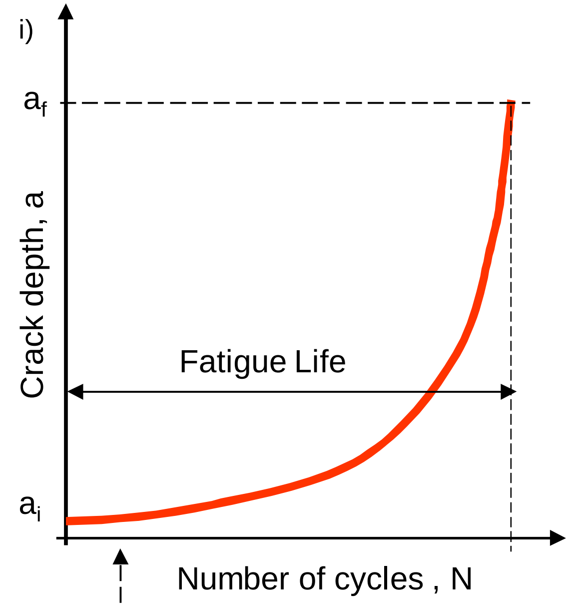
Integration of Paris' equation

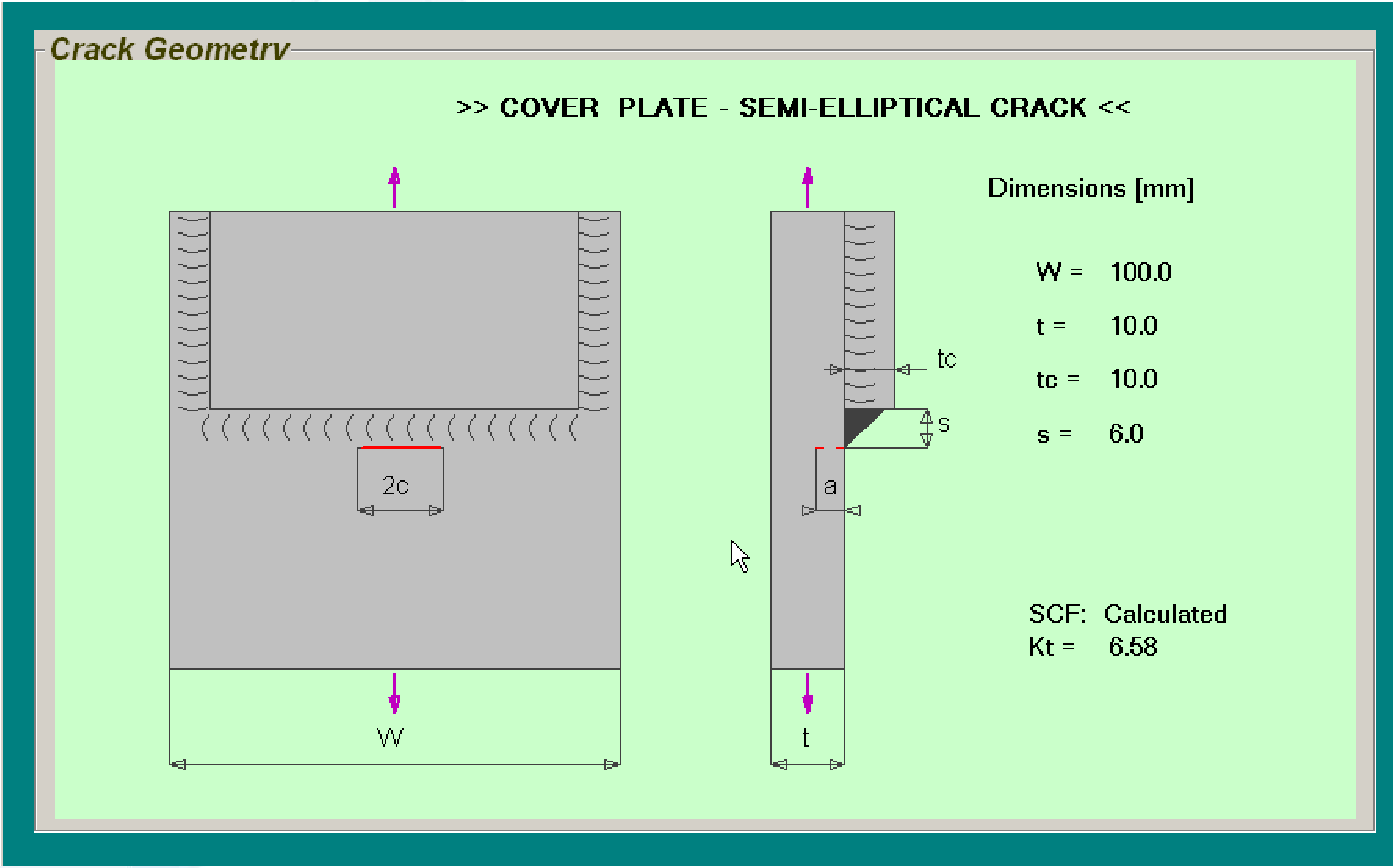
$$\Delta a_i = C\left(\Delta K_i\right)^m \Delta N_i$$

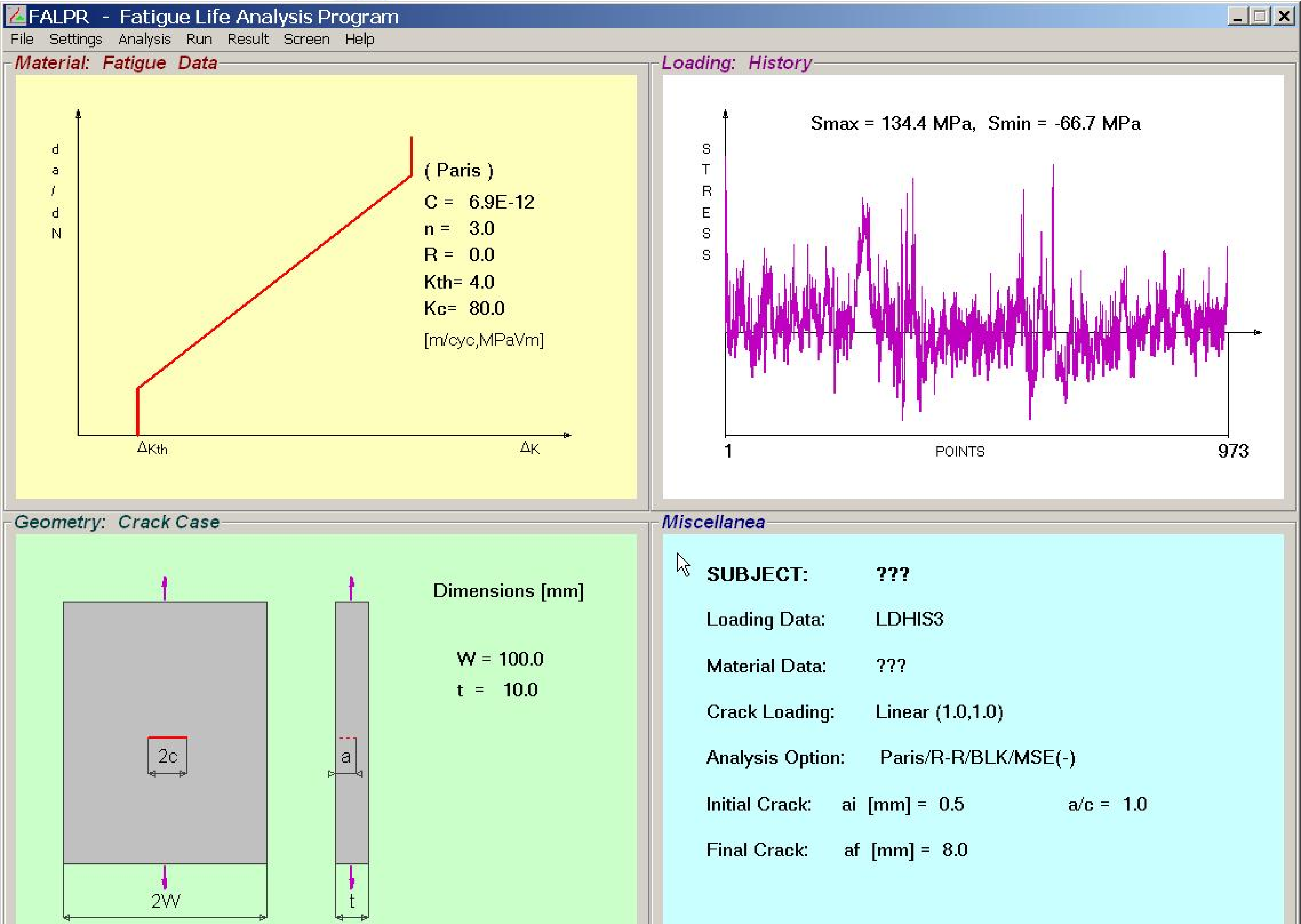
$$a_f = a_0 + \sum_{i=1}^{N} \Delta a_i$$

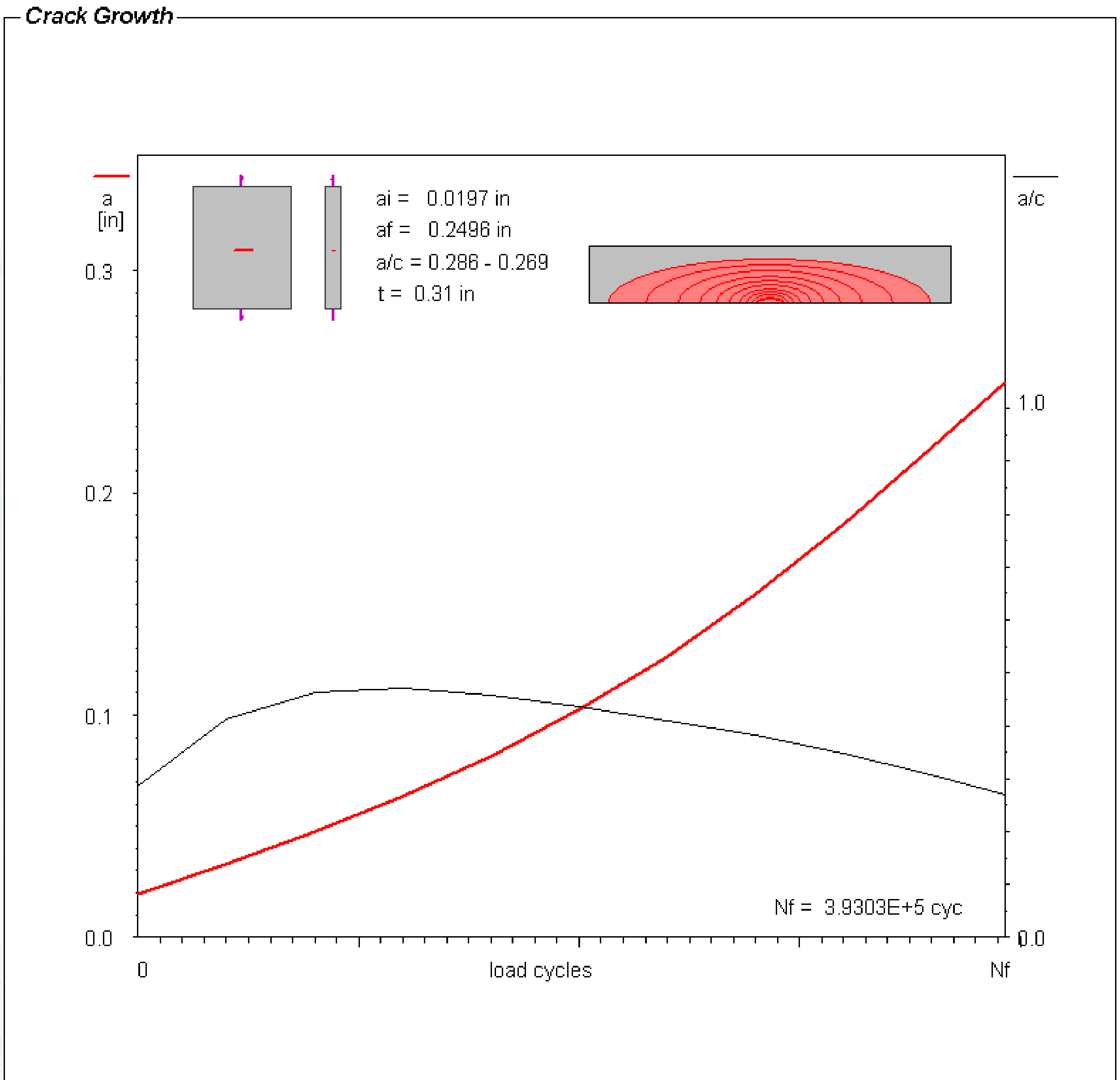
$$N = \sum \Delta N_i$$

i)




Crack Geometry
>> COVER PLATE - SEMI-ELLIPTICAL CRACK <<
Dimensions [mm]
W = 100.0
t = 10.0
tc = 10.0
s = 6.0
SCF: Calculated
Kt = 6.58
2c
a
tc
s
W
t


FALPR - Fatigue Life Analysis Program
File Settings Analysis Run Result Screen Help
Material: Fatigue Data
d a / d N
(Paris)
C = 6.9E-12
n = 3.0
R = 0.0
Kth= 4.0
Kc= 80.0
[m/cyc,MPaVm]
ΔKth
ΔK
Loading: History
Smax = 134.4 MPa, Smin = -66.7 MPa
STRESS
1
POINTS
973
Geometry: Crack Case
Dimensions [mm]
W = 100.0
t = 10.0
2c
a
2W
t
Miscellanea
SUBJECT: ???
Loading Data: LDHIS3
Material Data: ???
Crack Loading: Linear (1.0,1.0)
Analysis Option: Paris/R-R/BLK/MSE(-)
Initial Crack: ai [mm] = 0.5 a/c = 1.0
Final Crack: af [mm] = 8.0



Input Data

Loading: Spectrum

Smax = 3.0 ksi
Smin = -3.0 ksi
Levels: 1

Material: da/dN-data

A22-H
(Paris - in/cyc,ksi)

C = 2.98E-10
n = 3.02
R = 0.0
Kth = 3.19
Kc = 72.81

Analysis Options

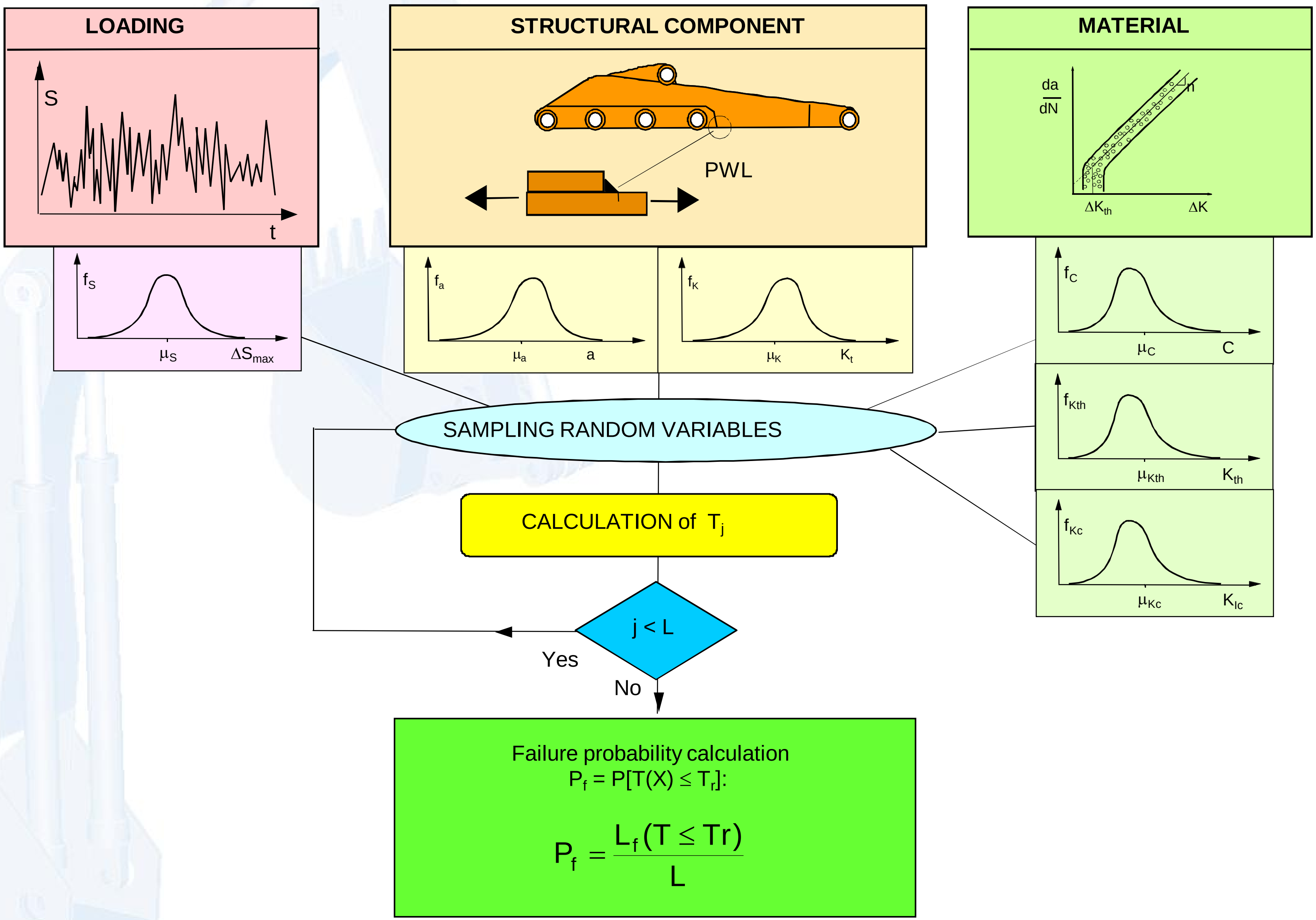
RNF/C-C/R(+)

Crack Loading

New2

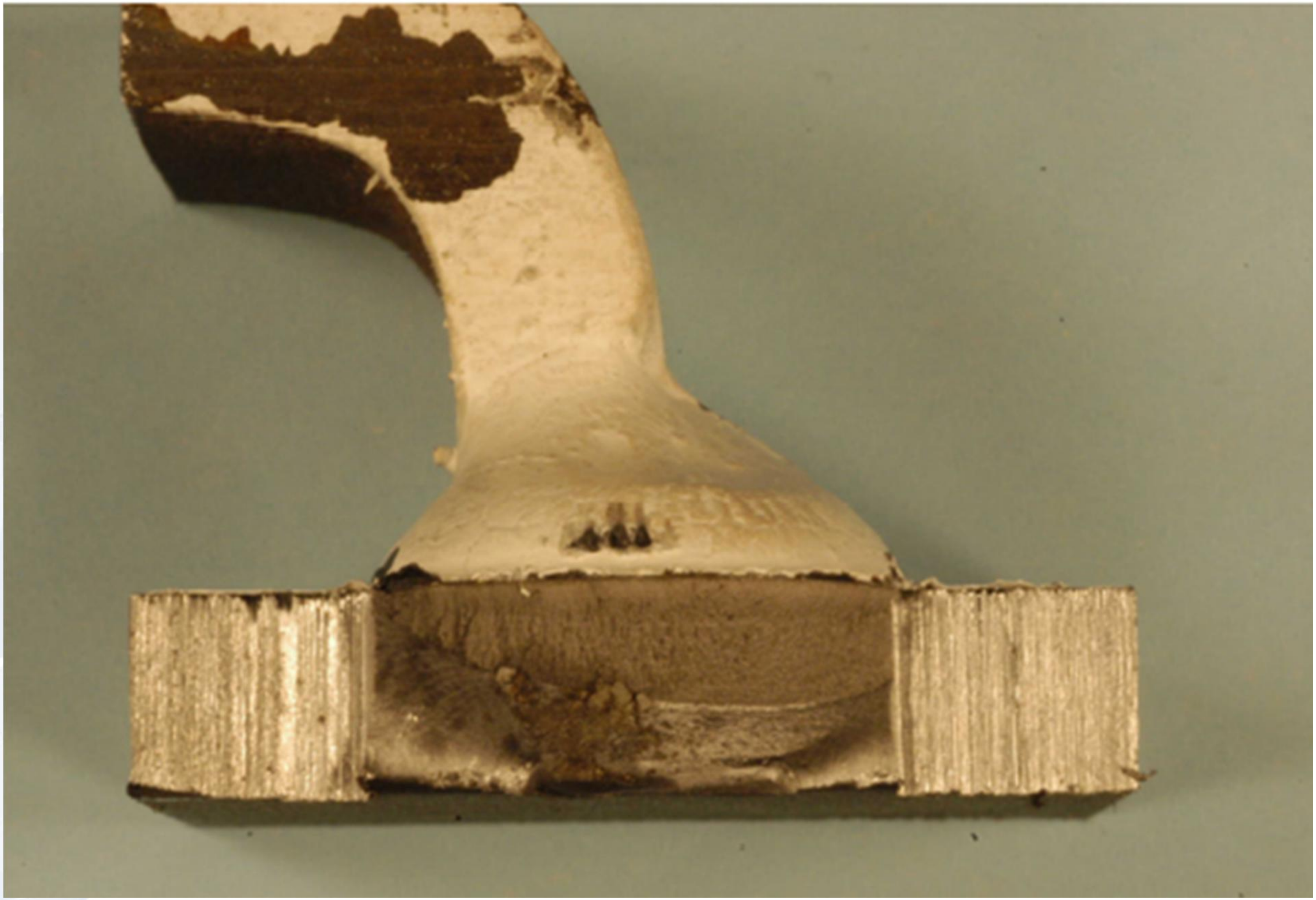
Welded Joint (Pos.1)

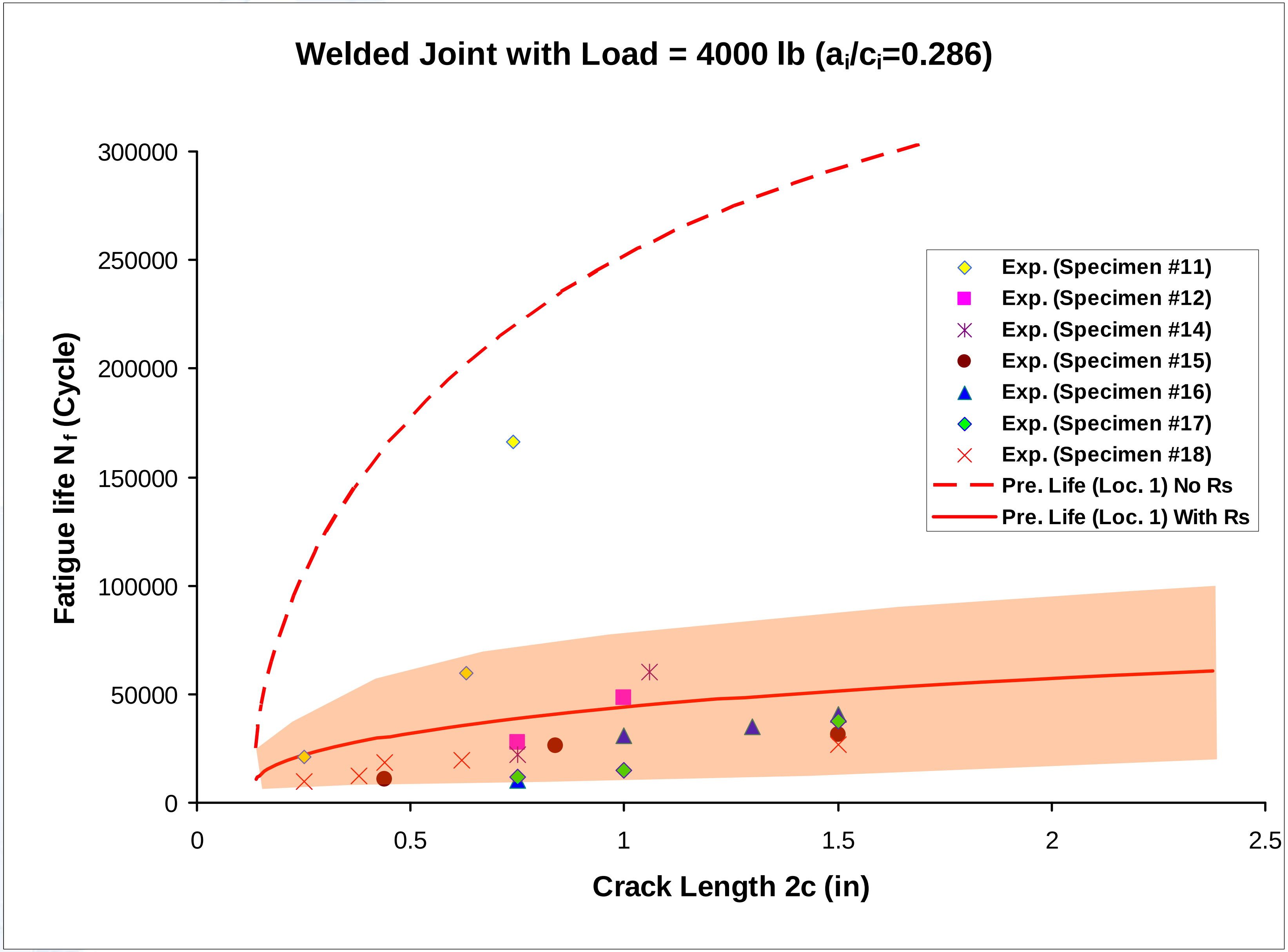
Probabilistic analysis using MC simulation



The FALPR statistical simulation flow chart for the analysis of fatigue crack growth

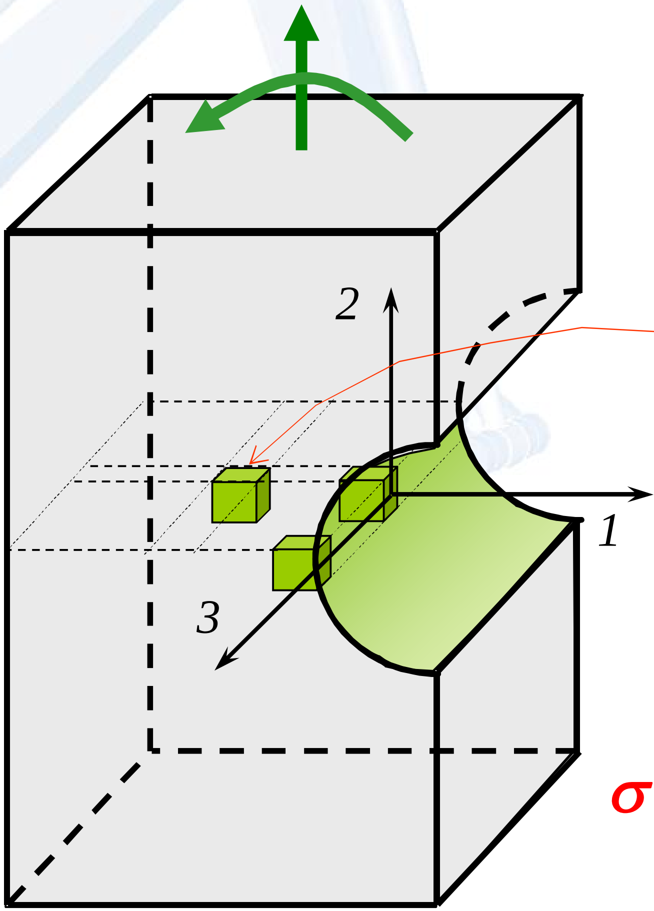
Irregular geometrical shape of a real fatigue crack

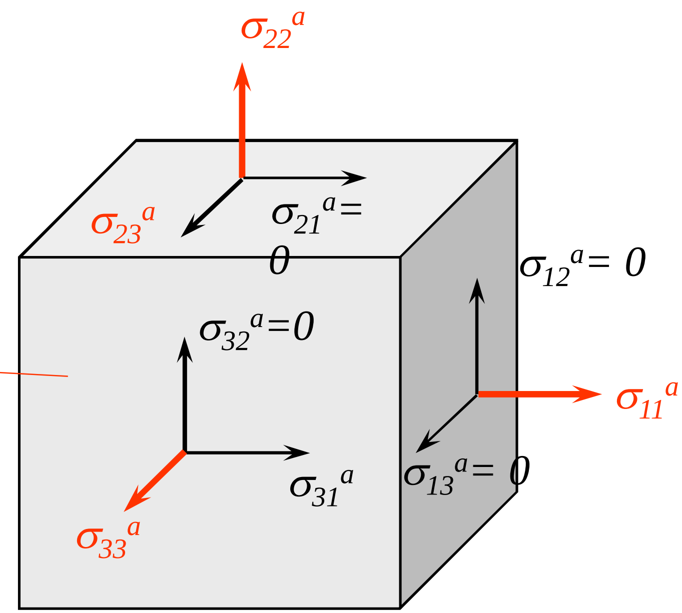



Welded Joint with Load = 4000 lb (a_i/c_i=0.286)
300000
250000
200000
150000
100000
50000
0
Fatigue life N_f (Cycle)
0
0.5
1
1.5
2
2.5
Crack Length 2c (in)
Exp. (Specimen #11)
Exp. (Specimen #12)
Exp. (Specimen #14)
Exp. (Specimen #15)
Exp. (Specimen #16)
Exp. (Specimen #17)
Exp. (Specimen #18)
Pre. Life (Loc. 1) No Rs
Pre. Life (Loc. 1) With Rs

Global and Local Approaches to Stress Analysis and Fatigue

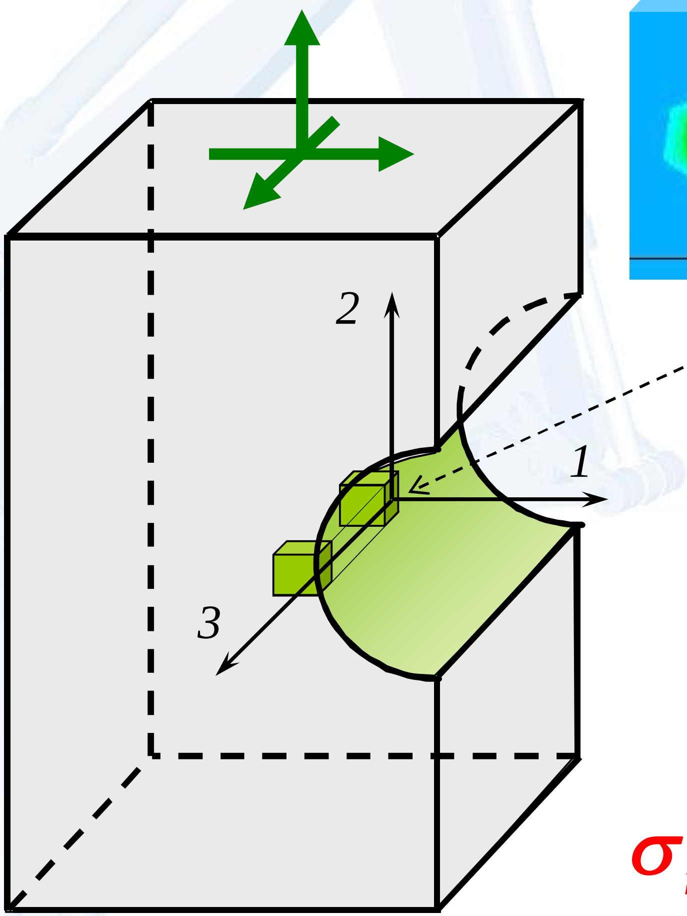
Stress state near the notch tip (on the symmetry line)

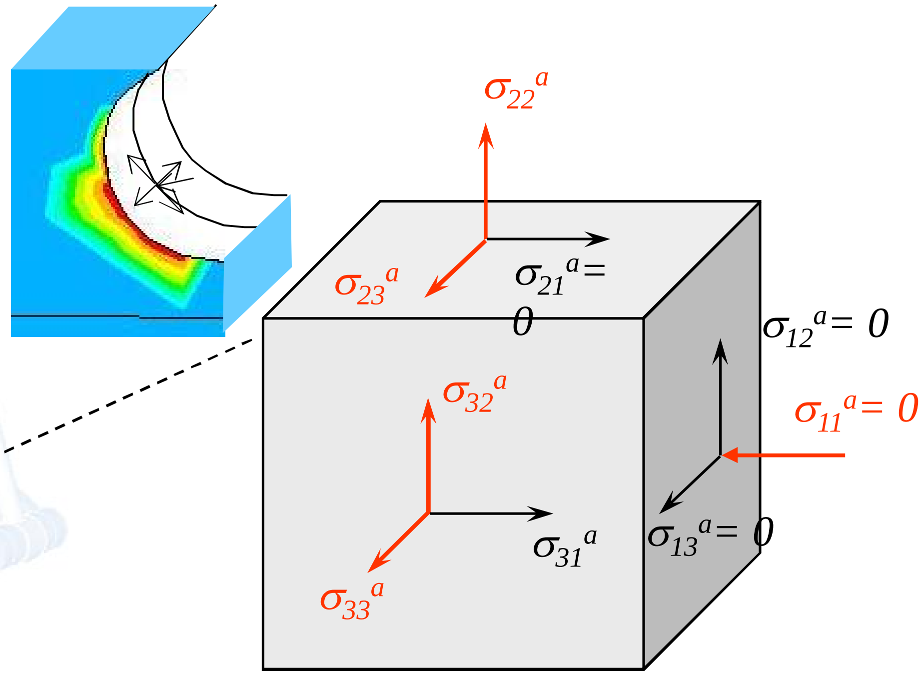




$$\sigma_{ij}^{a} = \begin{bmatrix} \sigma_{11}^{a} & 0 & 0 \\ 0 & \sigma_{22}^{a} & 0 \\ 0 & 0 & \sigma_{33}^{a} \end{bmatrix}$$

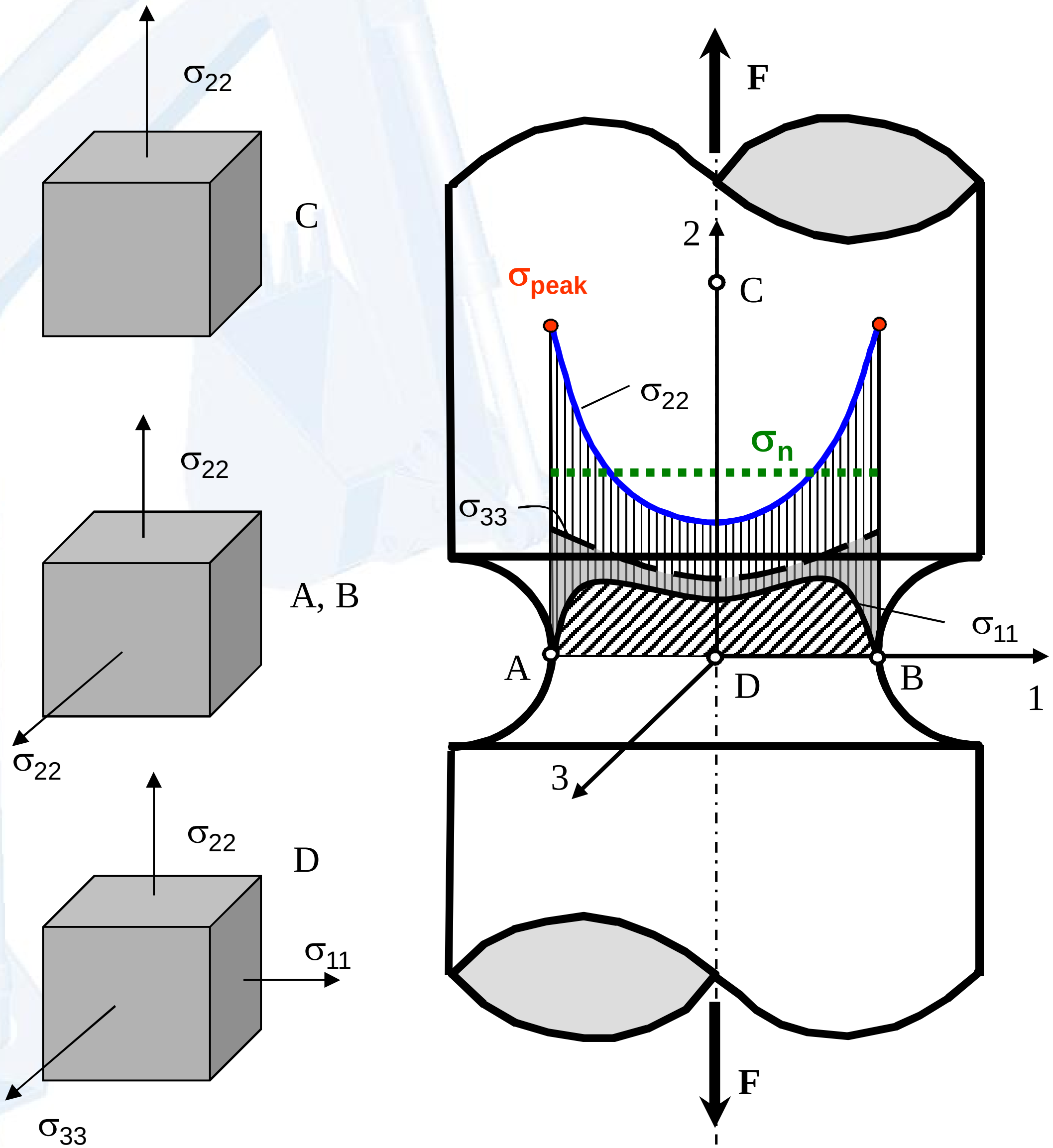
Stress state in the disk at the blade-disk interface





$$\sigma_{ij}^{a} = \begin{bmatrix} \sigma_{11}^{a} & 0 & 0 \\ 0 & \sigma_{22}^{a} & \sigma_{23}^{a} \\ 0 & \sigma_{32}^{a} & \sigma_{33}^{a} \end{bmatrix}$$

Stresses concentration in axis-symmetric notched body

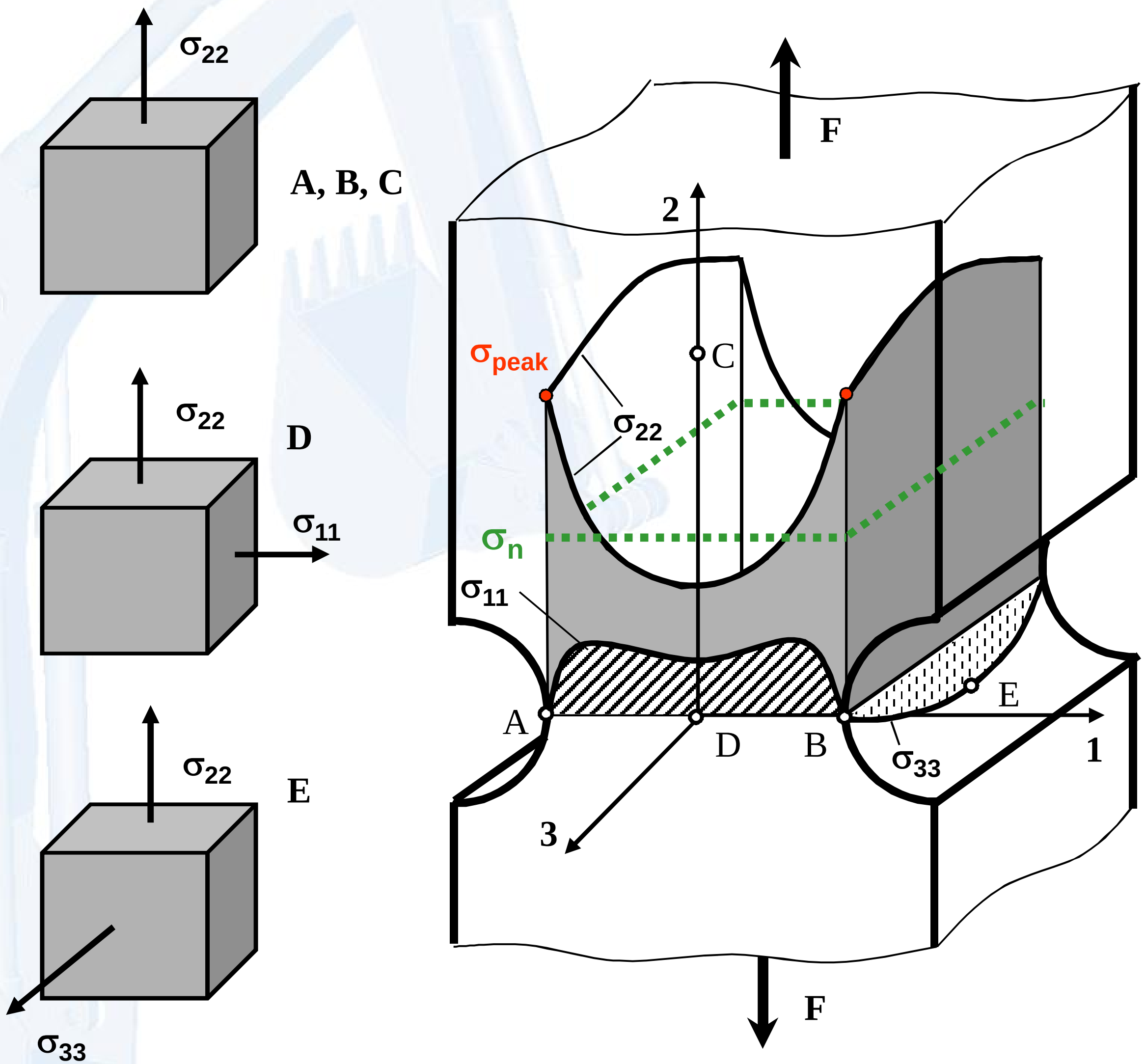


$$\sigma_n = \frac{F}{A_{net}}$$

and

$$\sigma_{peak} = K_t \sigma_n$$

Stresses concentration in a prismatic notched body

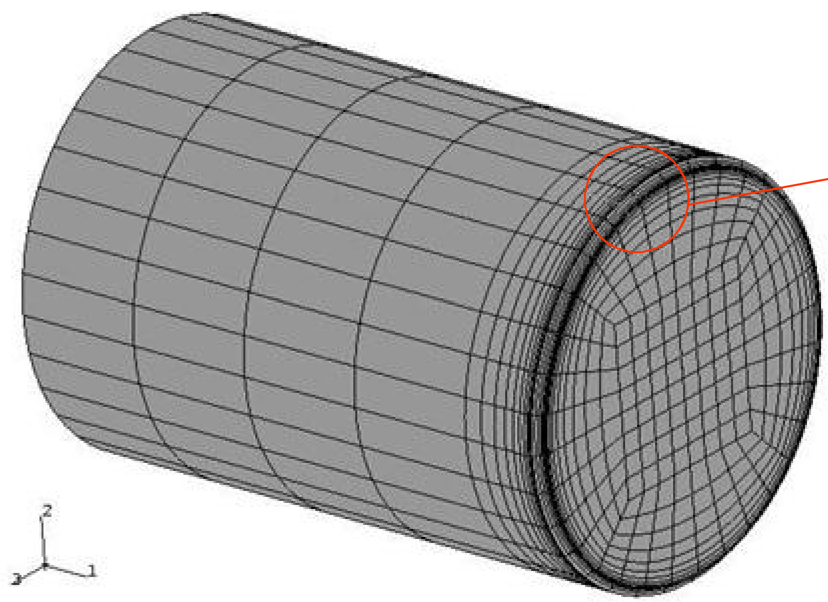


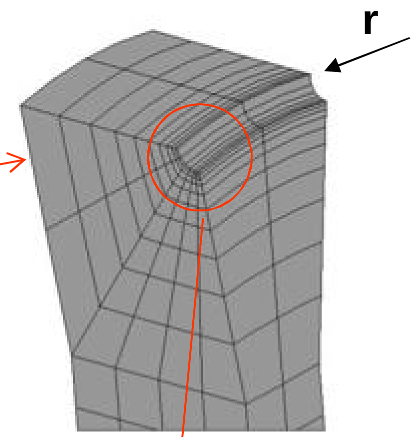
$$\sigma_n = \frac{F}{A_{net}}$$

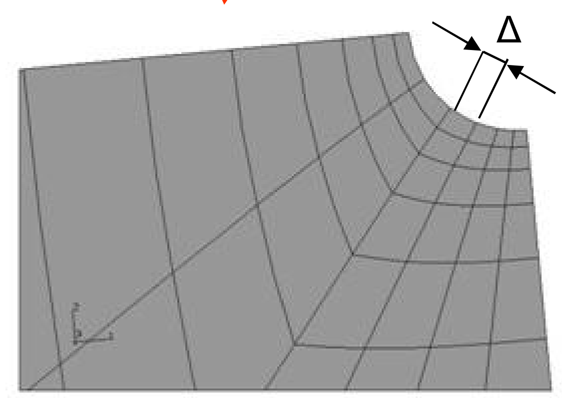
and

$$\sigma_{peak} = K_t \sigma_n$$

How to get the nominal stress from the Finite Element Method stress data?

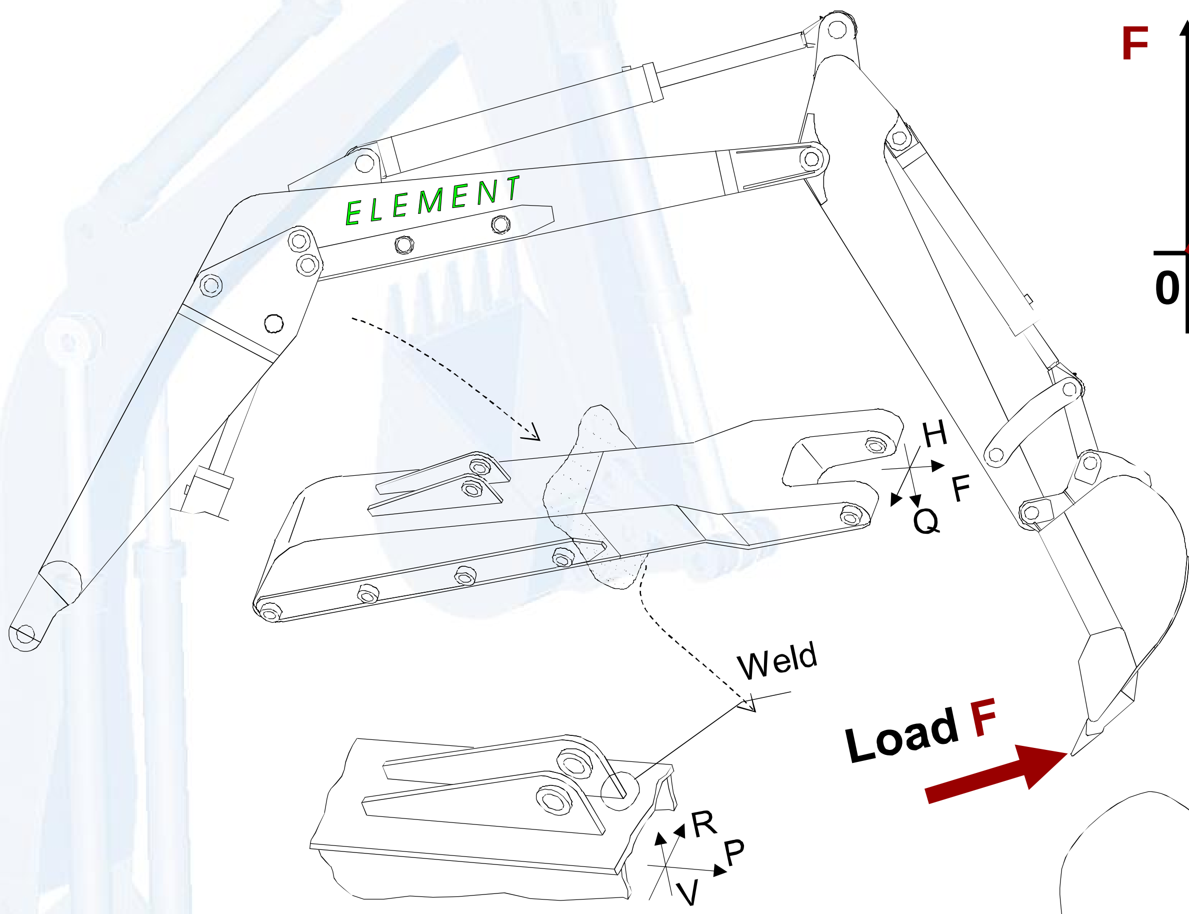


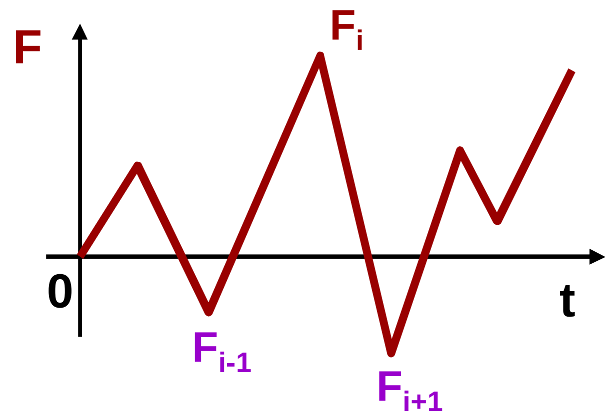




The smallest optimum element size $\Delta < \frac{1}{4}r$!!

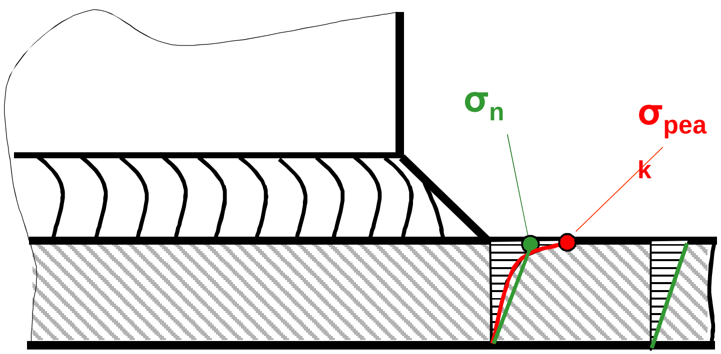
Loads and stresses in a structure





$$\sigma_{peak,i} = f\left(F_i\right); \quad f\left(F\right) = ?$$

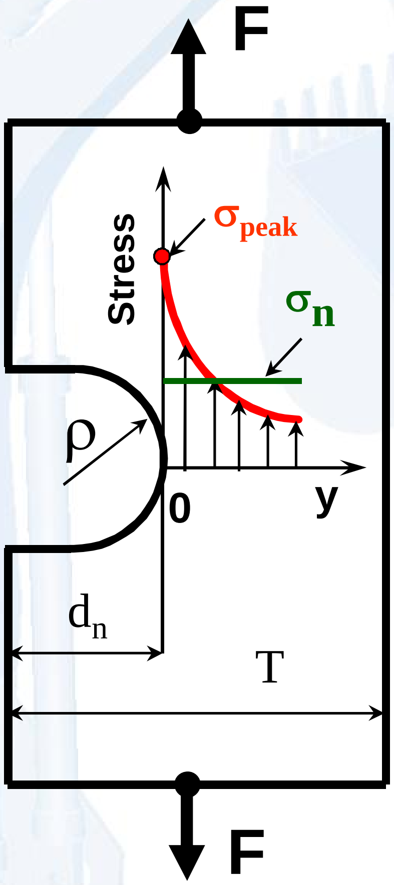
$$\sigma_{n,i} = g\left(F_i\right); \quad g\left(F\right) = ?$$



Loads and Stresses

The load, the nominal stress, the local peak stress and the stress concentration factor

Axial load – linear elastic analysis



$$\sigma_n = \frac{F}{A_{net}};$$

$$\sigma_n = k_F F; \qquad \sigma_{peak} = h_F F;$$

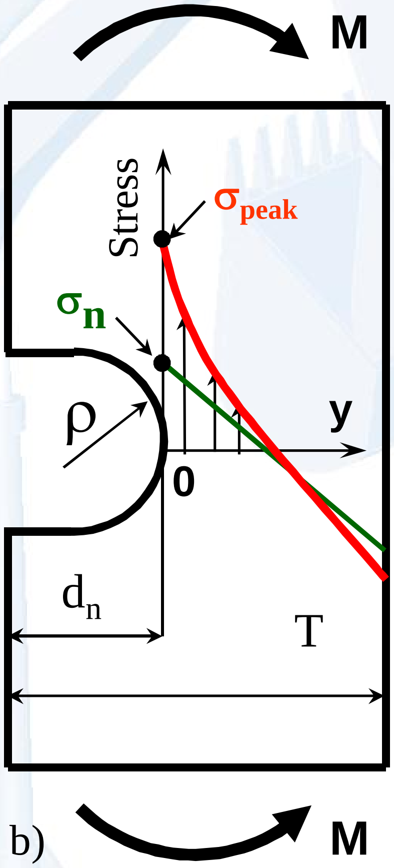
Analytical, FEM | Hndbk

$$k_F = \frac{1}{A_{net}}; \qquad h_F = \frac{\sigma_{peak}}{F}; \qquad h_F = k_F K_t;$$

$$\sigma_{n,i} = k_F F_i; \qquad \sigma_{peak,i} = h_F F_i;$$

Loads and Stresses

The load, the nominal stress, the local peak stress and the stress concentration factor



Bending load – linear elastic analysis

$$\sigma_n = \frac{M \cdot c_{net}}{I_{net}};$$

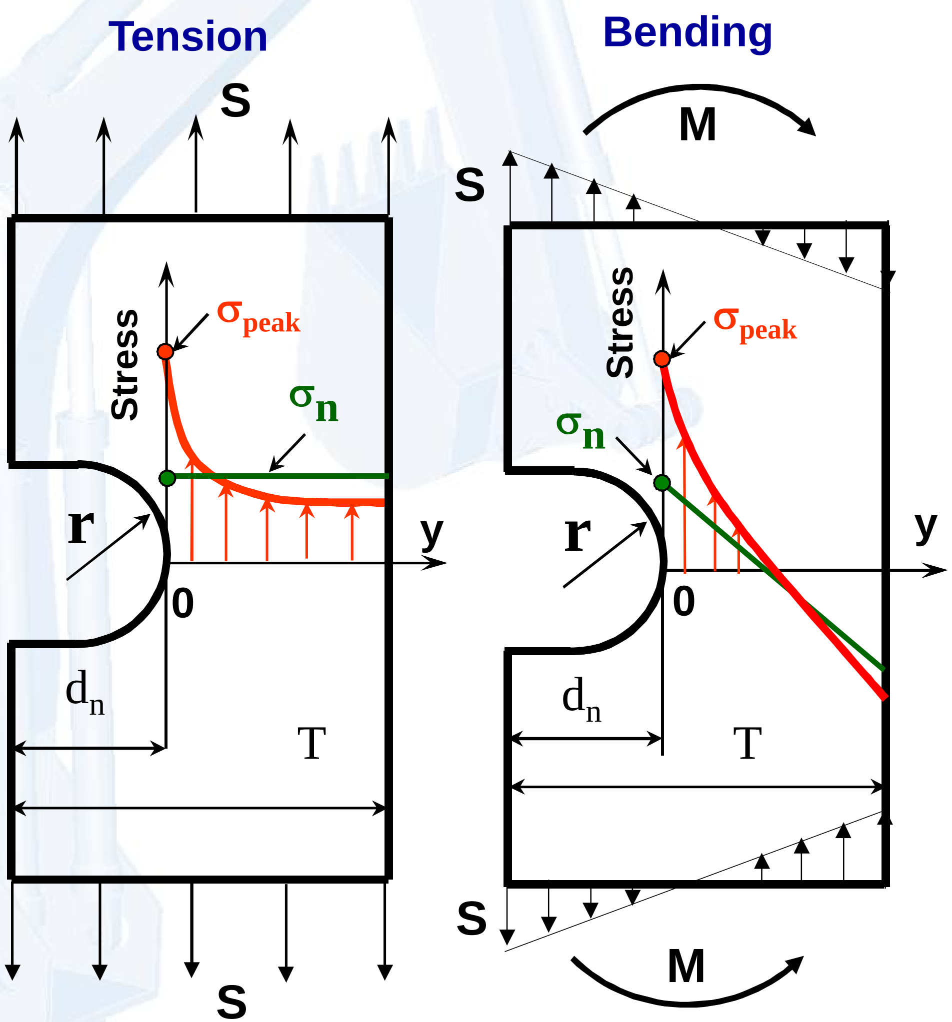
$$\sigma_n = k_M M;$$

$$k_M = \frac{c_{net}}{I_{net}}; \Rightarrow \sigma_{n,i} = k_M M_i;$$

$$\sigma_{peak,i} = h_M M_i \text{ or } \sigma_{peak,i} = k_M M_i \cdot K_t^b$$

Stress Concentration Factors in Fatigue Analysis

The nominal stress and the stress concentration factor in simple load/geometry configurations



Simple axial load

$$\sigma_n = \frac{P}{A_{net}} \quad or \quad S = \frac{P}{A_{gross}}$$

Pure bending load

$$\sigma_n = \frac{M \cdot c_{net}}{I_{net}} \quad or \quad S = \frac{M \cdot c_{gross}}{I_{gross}}$$

$$\underbrace{K_t = \frac{\sigma_{peak}}{\sigma_n}}_{\text{net } K_t} \quad or \quad \underbrace{K_t = \frac{\sigma_{peak}}{S}}_{\text{gross } K_t}$$

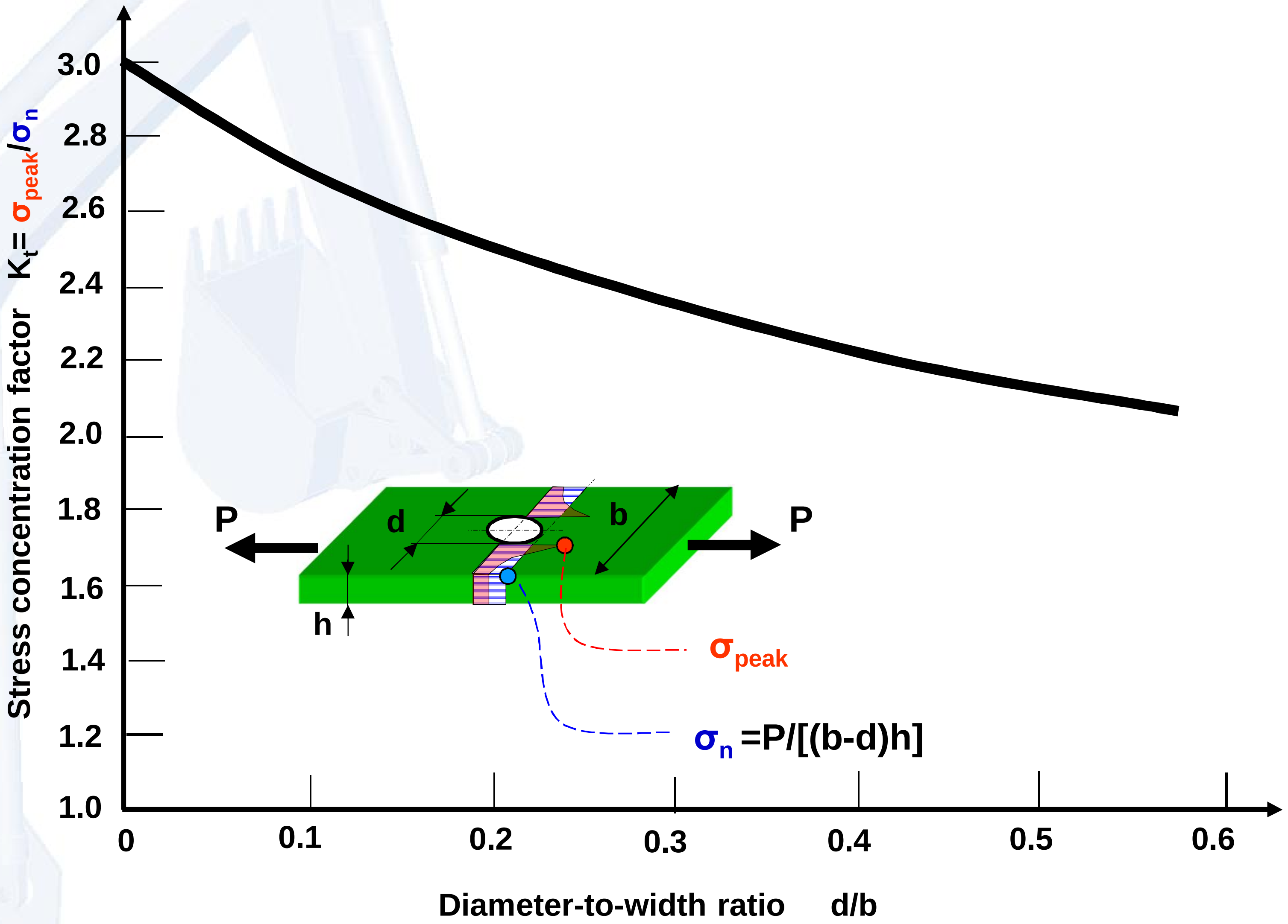
K_t – stress concentration factor (net or gross, net $K_t \neq$ gross K_t !!)

σ_{peak} – stress at the notch tip

σ_n - net nominal stress

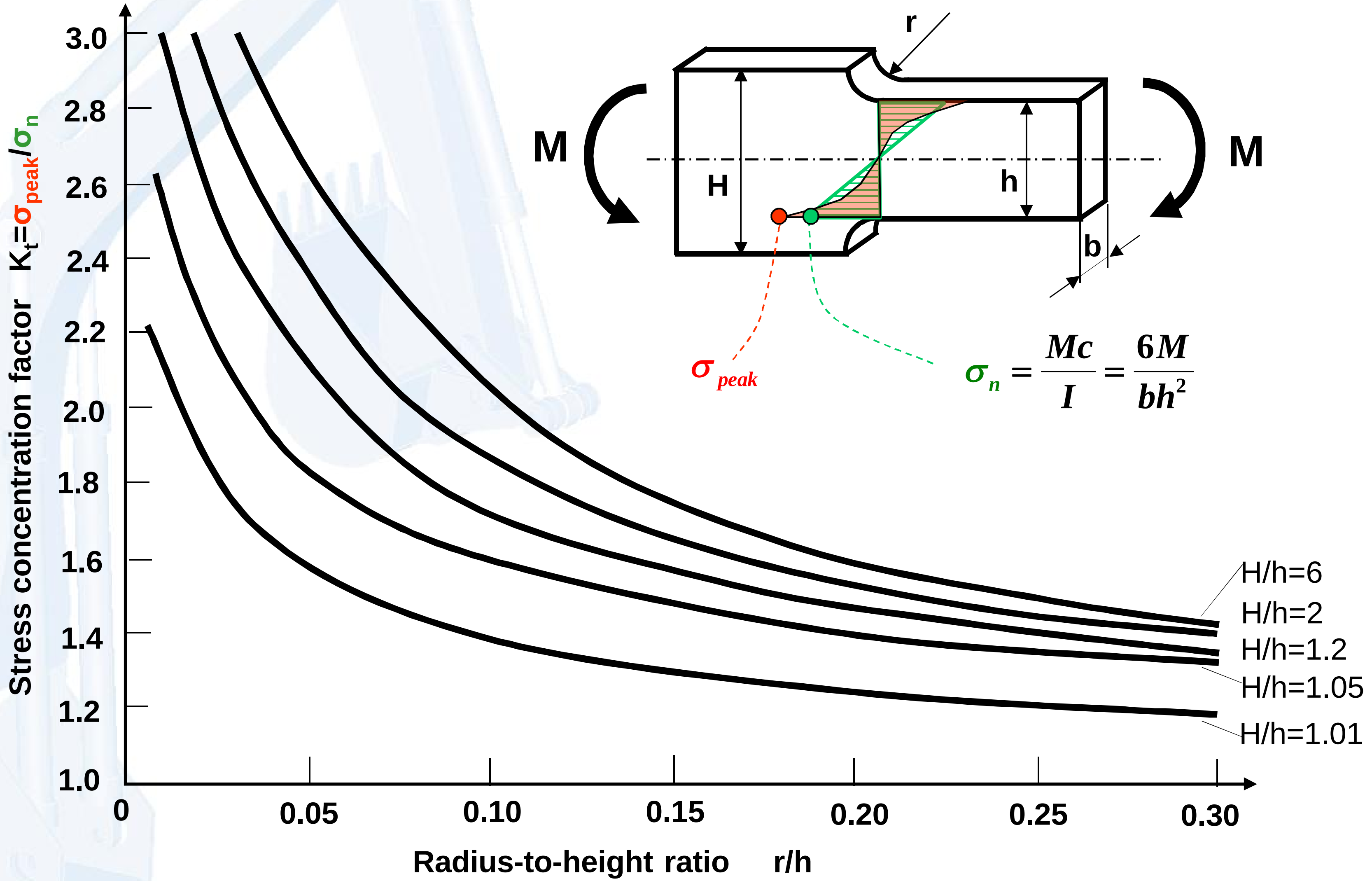
S - gross nominal stress

Stress concentration factors for notched machine components



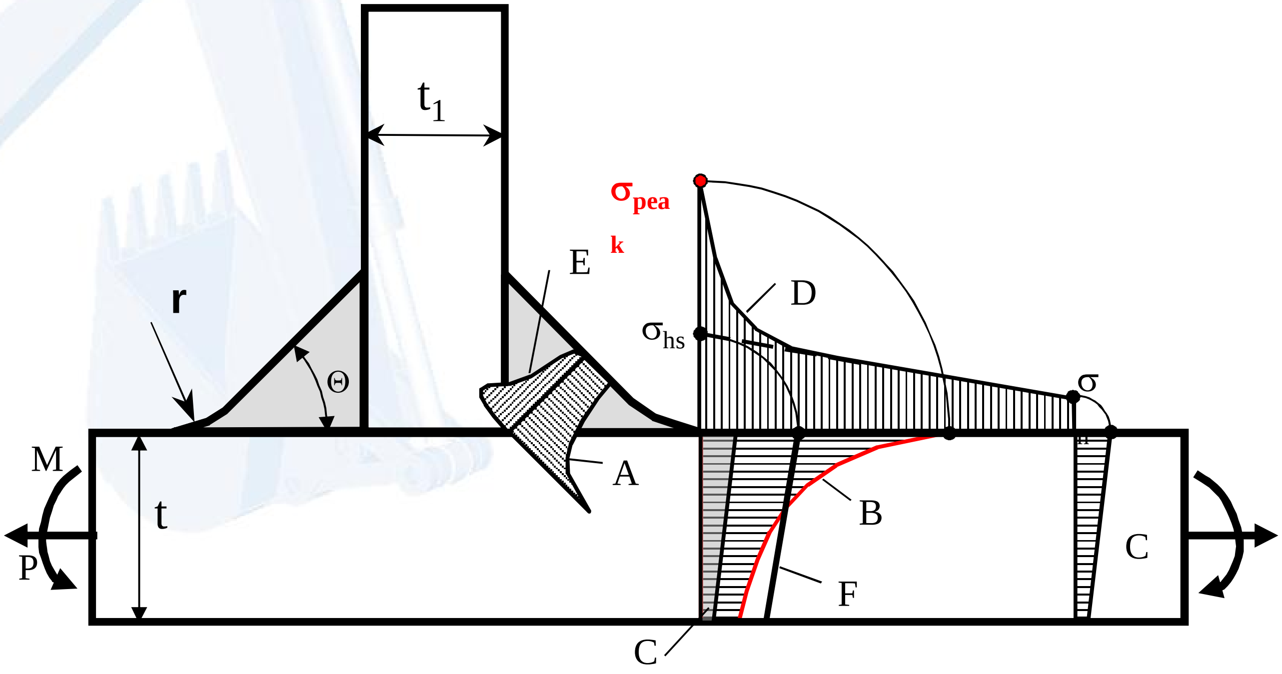
(B.J. Hamrock et. al., ref.(26))

Stress concentration factors for notched machine components



(B.J. Hamrock et. al.

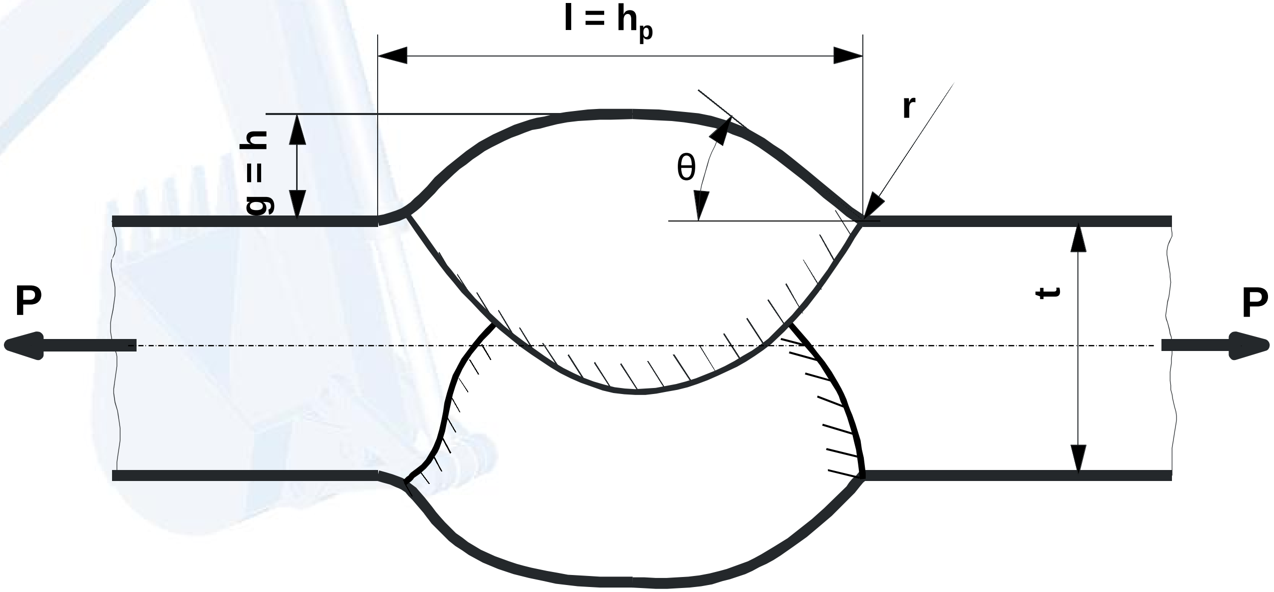
Stress concentration & stress distributions in weldments



Various stress distributions in a T-butt weldment with transverse fillet welds;

- Normal stress distribution in the weld throat plane (A),
- Through the thickness normal stress distribution in the weld toe plane (B),
- Through the thickness normal stress distribution away from the weld (C),
- Normal stress distribution along the surface of the plate (D),
- Normal stress distribution along the surface of the weld (E),
- Linearized normal stress distribution in the weld toe plane (F).

Stress concentration factor for a butt weldment under axial loading



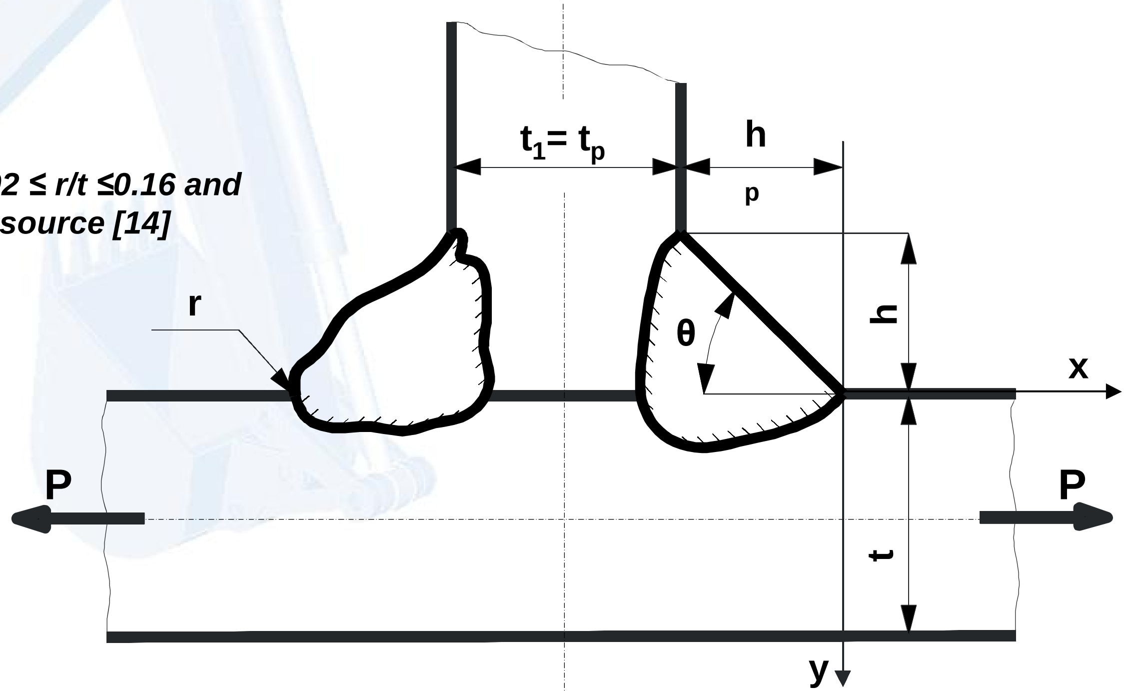
$$K_t^{ten} = 1 + \frac{1 - \exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1 - \exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times 2\left[\frac{1}{2.8\left(\frac{W}{t}\right) - 2} \times \frac{h}{r}\right]^{0.65}$$

$$where:\ W = t + 2h + 0.6h_p$$

Range of application - reasonably designed weldments, (K.Iida and T. Uemura, ref. 14)

Stress concentration factor for a T-butt weldment under tension load; *(non-load carrying fillet weld)*

Validated for : $0.02 \le r/t \le 0.16$ and $30^o \le \theta \le 60^o$, source [14]



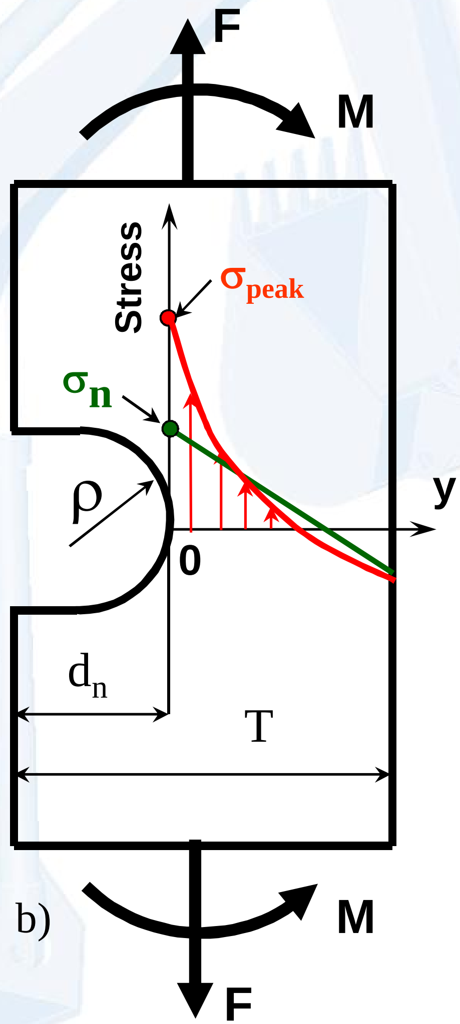
$$K_t^t = 1 + \frac{1-\exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1-\exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times \left[\frac{1}{2.8\left(\frac{W}{t}\right)-2} \times \frac{h}{r}\right]^{0.65}$$

$$where : W = (t+2h) + 0.3(t_p + 2h_p)$$

Cyclic Loads and Cyclic Stress Patterns (histories) in Engineering Objects

Loads and Stresses

The load, the nominal stress, the local peak stress and the stress concentration factor



Simultaneous axial and bending load

$$\sigma_{n,i} = k_F F_i + k_M M_i;$$

$$\sigma_{peak,i} = k_F F_i \cdot K_t^t + k_M M_i K_t^b;$$

or

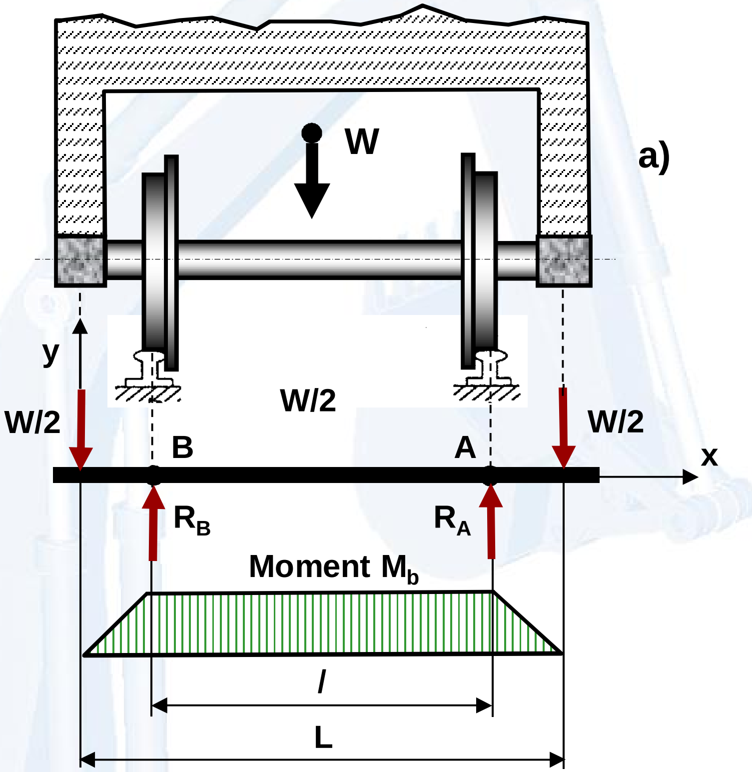
$$\sigma_{peak,i} = h_F F_i + h_M M_i;$$

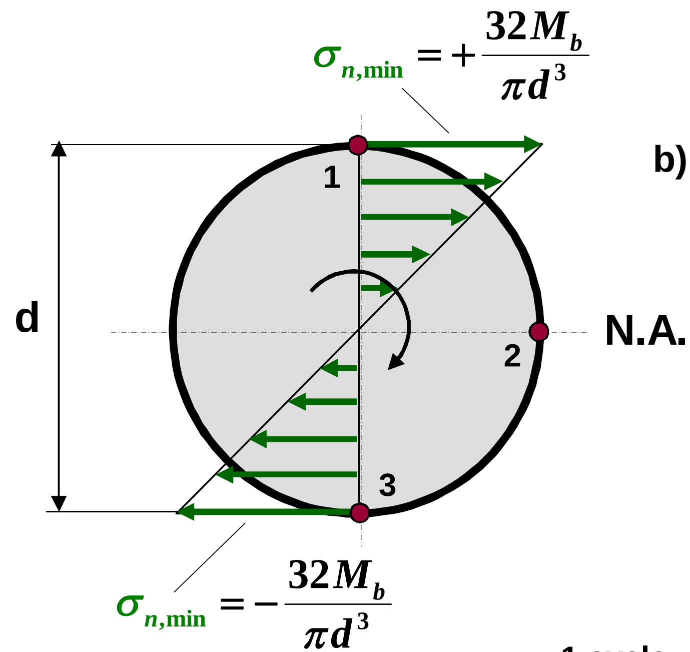
$k_F,\ k_M;$ **From simple analytical stress analysis**

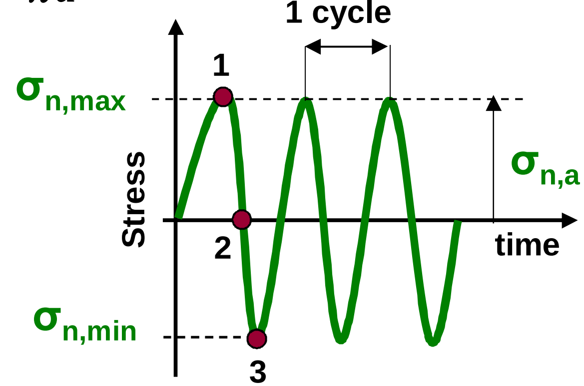
$K_t^t,\ K_t^b;$ **From stress concentration handbooks**

$h_F,\ h_M;$ **From detail FEM analysis**

The load W and the nominal stress σ_n in an railway axle





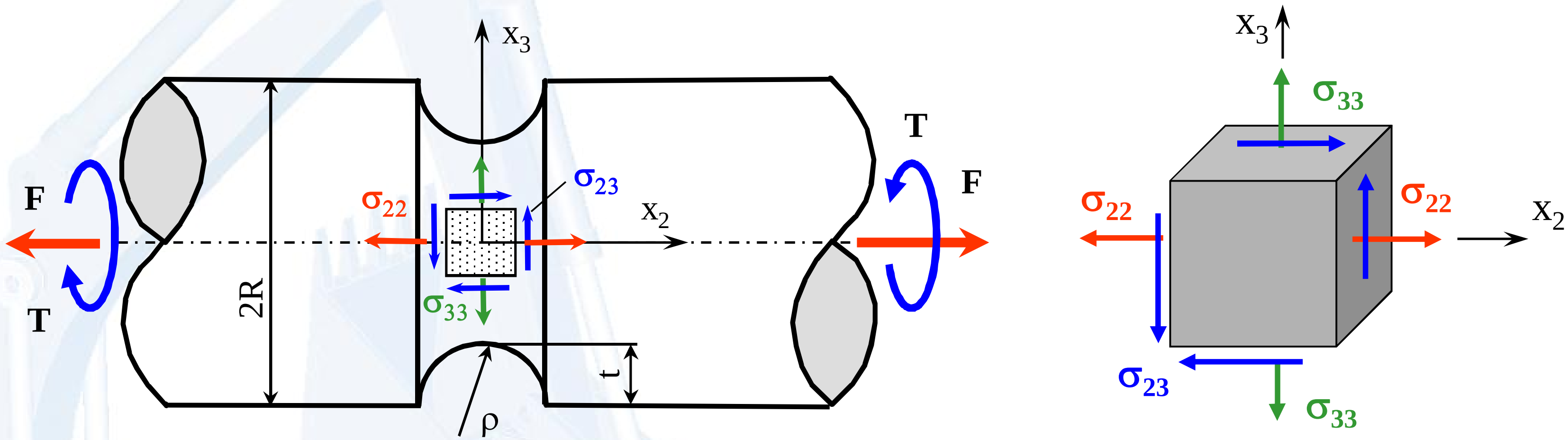


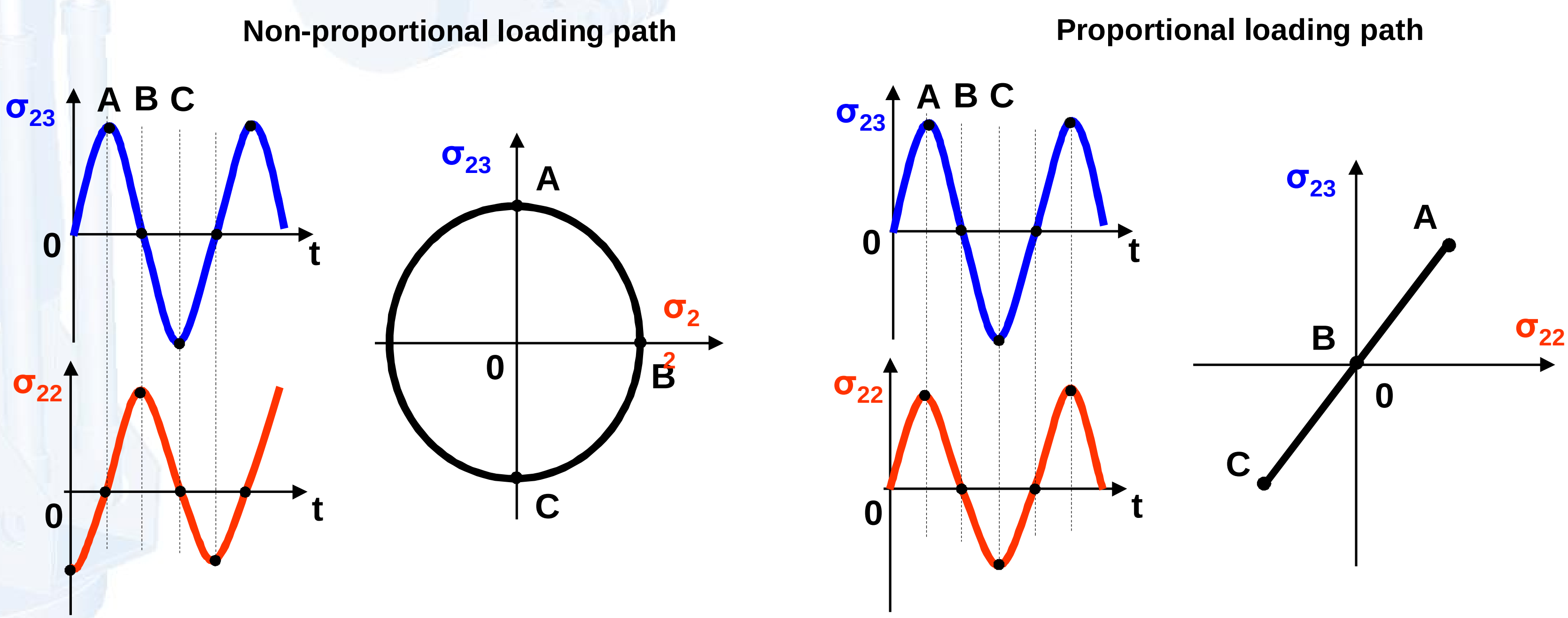
$$S = \sigma_n = \sigma_{peak} = \frac{M_b c}{I} = \pm\frac{32M_b}{\pi d^3};$$

$$M_b = \frac{W}{4}(L+l); \quad c = \frac{d}{2}; \quad I = \frac{\pi d^3}{64};$$

Note! **In the case of smooth components, such as the railway axle, the nominal stress and the local peak stress are the same!**

Fluctuations and complexity of the stress state at the notch tip





How to establish the nominal stress history?

a) The analytical or FE analysis should be carried out for one characteristic load magnitude, i.e. P=1, M_b =1, T=1 in order to establish the proportionality factors, k_P, k_M, and k_T such that:

$$\sigma_n^P = k_P \cdot P; \quad \sigma_n^M = k_M \cdot M_b; \quad \tau_n^T = k_T \cdot T$$

b) The peak and valleys of the nominal stress history $\sigma_{n,,i}$ are determined by scaling the peak and valleys load history **P_i, $M_{b,I}$** and **T_i** by appropriate proportionality factors k_P, k_M, and k_T such that:

$$\sigma_{n,i}^P = k_P \cdot P_i \quad \sigma_{n,i}^M = k_M \cdot M_{b,i}; \quad \tau_{n,i}^T = k_T \cdot T_i$$

c) In the case of proportional loading the normal peak and valley stresses can be added and the resultant nominal normal stress history can be established. Because all load modes in proportional loading have the same number of simultaneous reversals the resultant history has also the same number of resultant reversals as any of the single mode stress history.

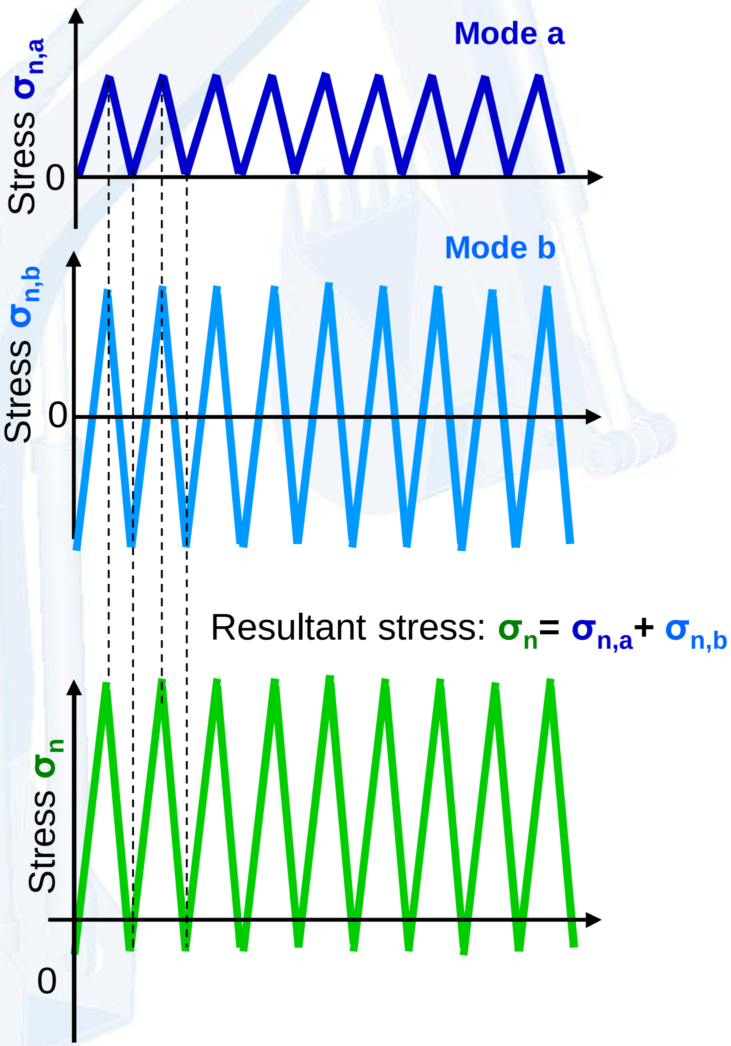
$$\sigma_{n,i} = k_P \cdot P_i + k_M \cdot M_{b,i};$$

d) In the case of non-proportional loading the normal stress histories (and separately the shear stresses) have to be added as time dependent processes. Because each individual stress history has different number of reversals the number of reversals in the resultant stress history can be established after the final superposition of all histories.

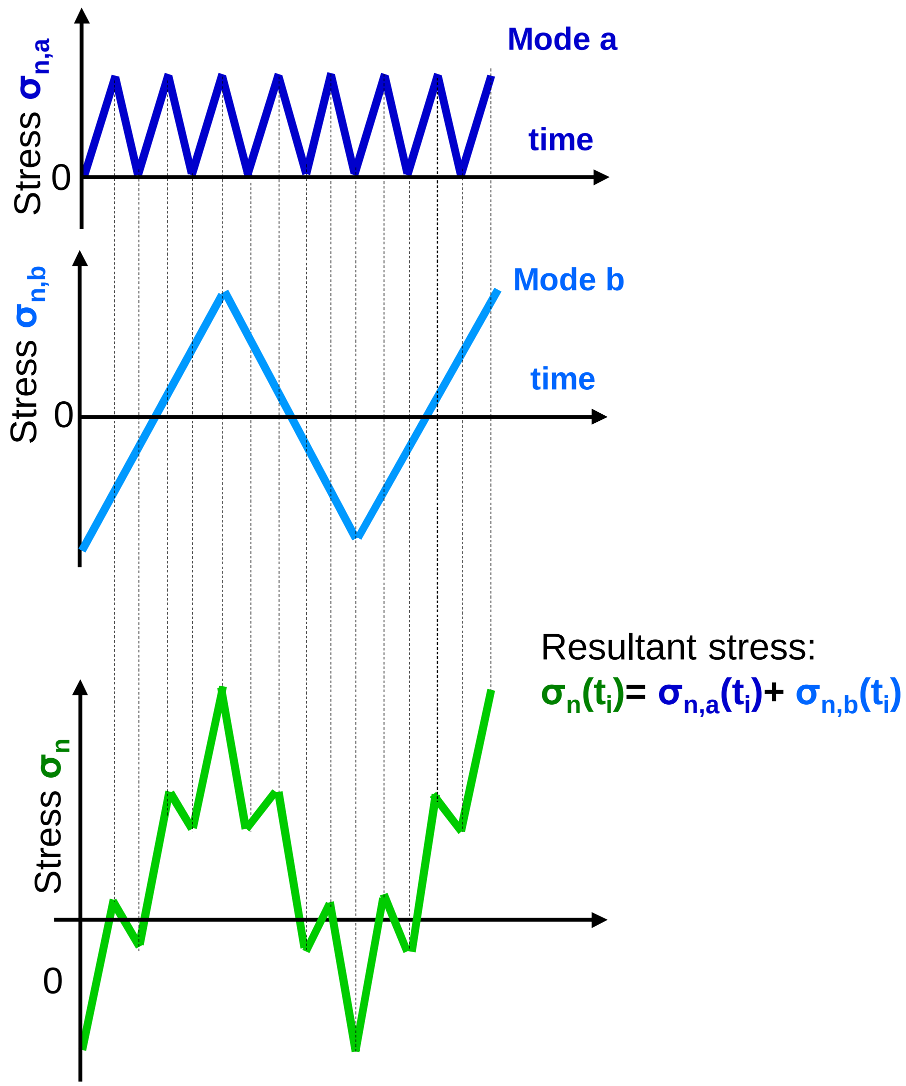
$$\sigma_n(t_i) = k_P \cdot P(t_i) + k_M \cdot M_b(t_i)$$

Superposition of nominal stress histories induced by two independent loading modes

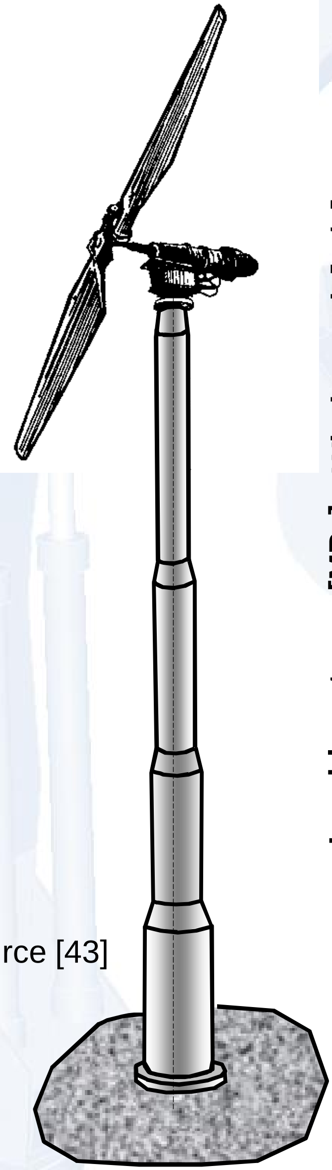
Two proportional modes of loading



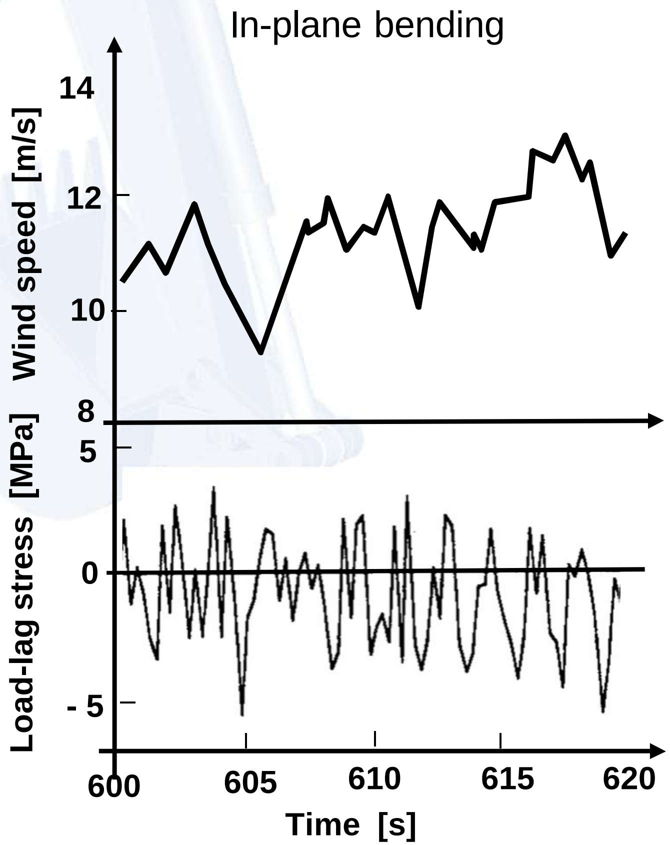
Two non-proportional modes of loading

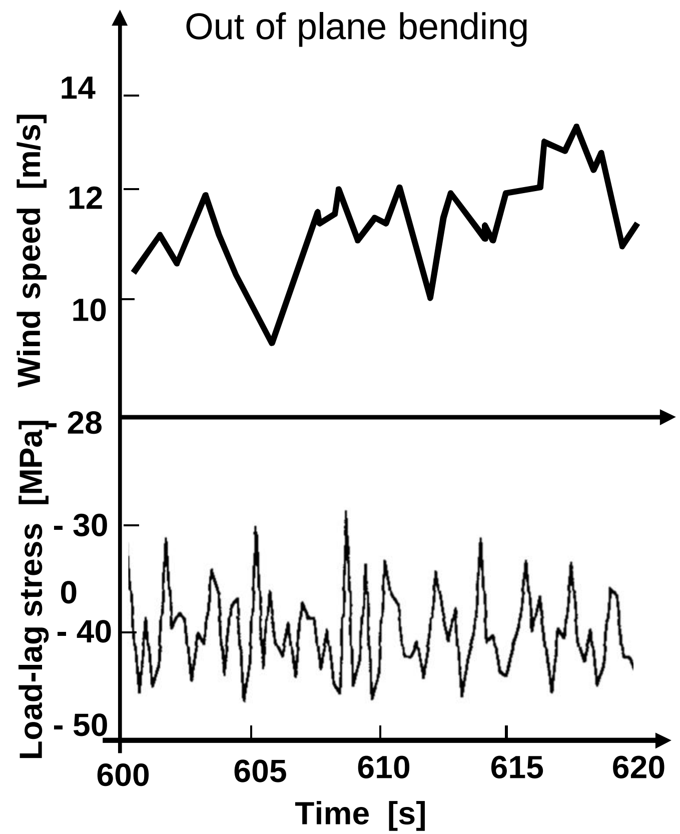


Wind load and stress fluctuations in a wind turbine blade



Source [43]

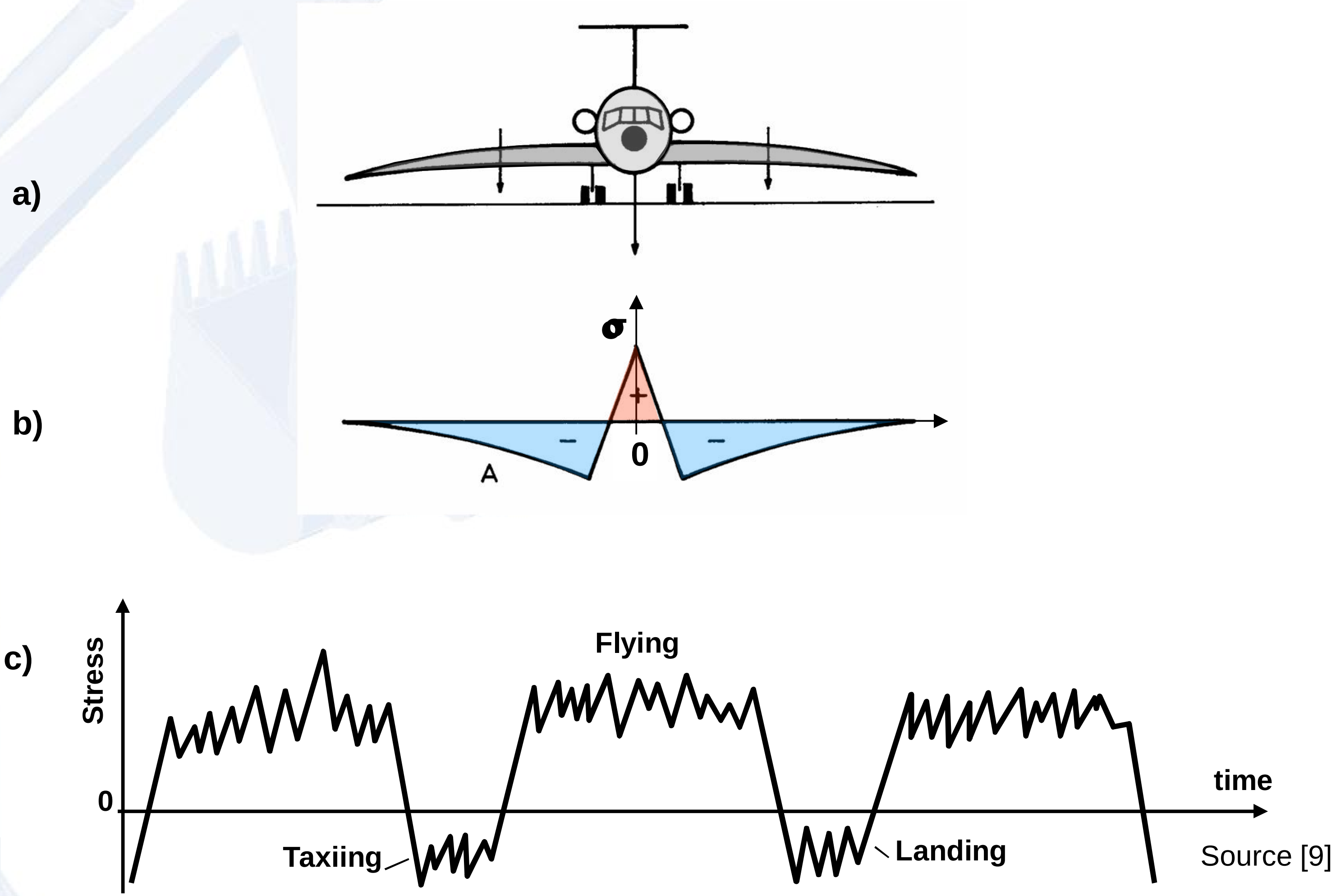




Note! One reversal of the wind speed results in several stress reversals

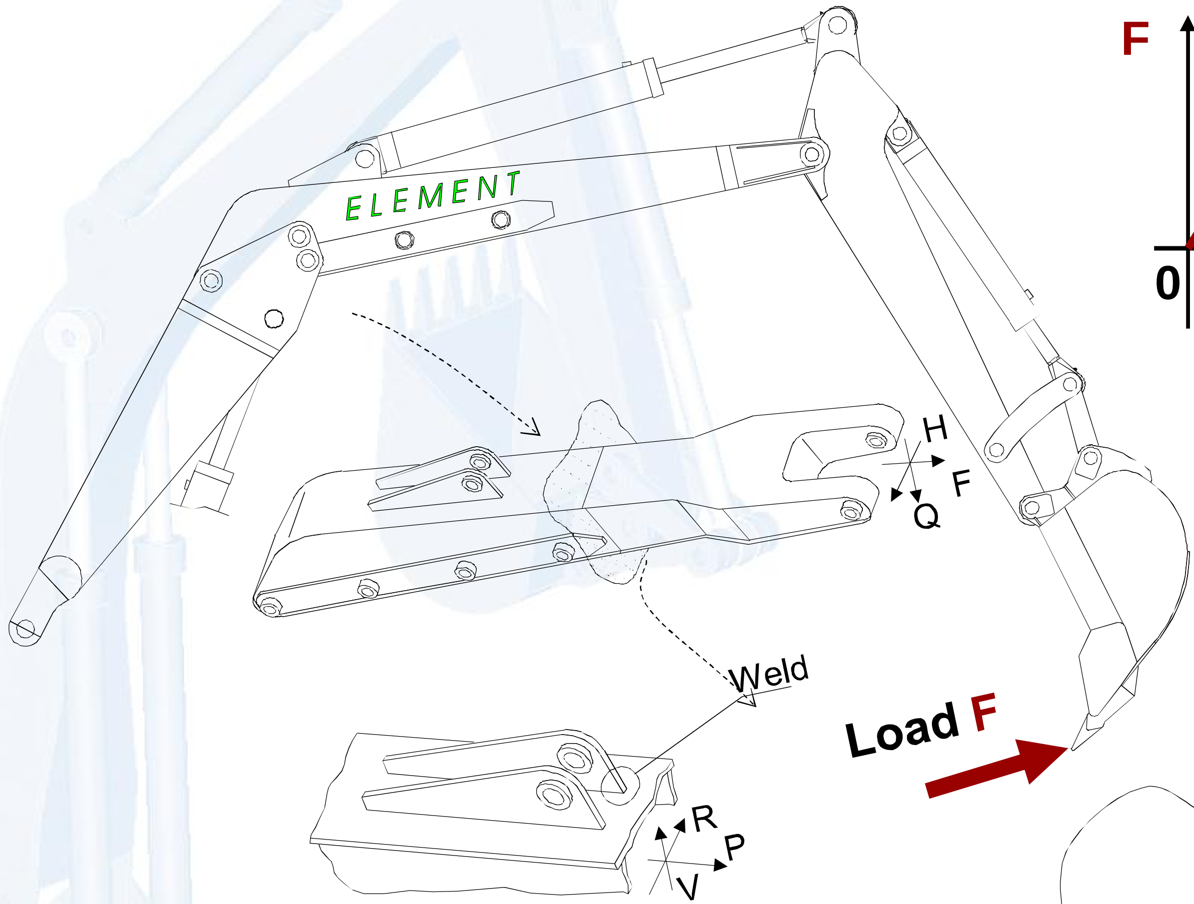
Wind speed fluctuations + Blade vibrations ⟶ Stress fluctuations

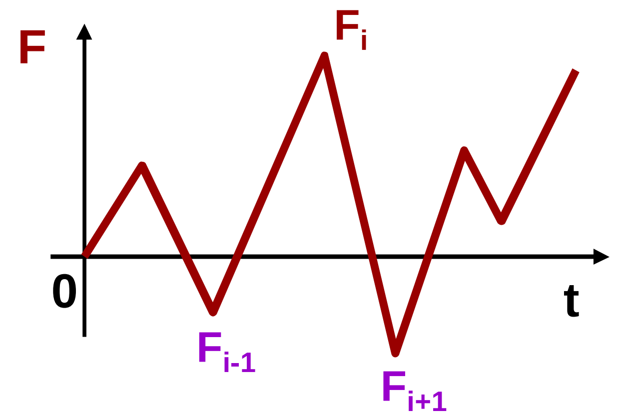
Characteristic load/stress history in the aircraft wing skin



a) Ground loads on the wings, b) Distribution of the wing bending moment induced by the ground load, c) Stress in the lower wing skin induced by the ground and flight loads

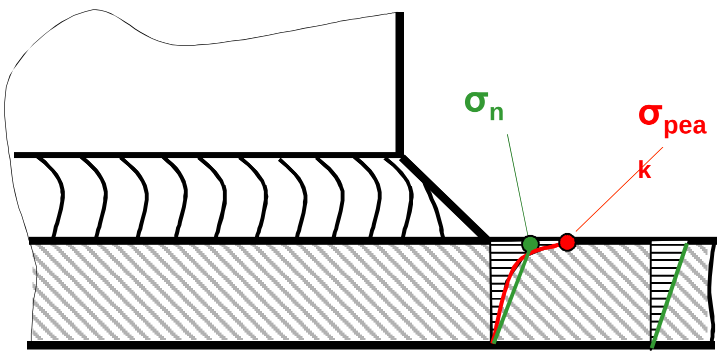
Loads and stresses in a structure





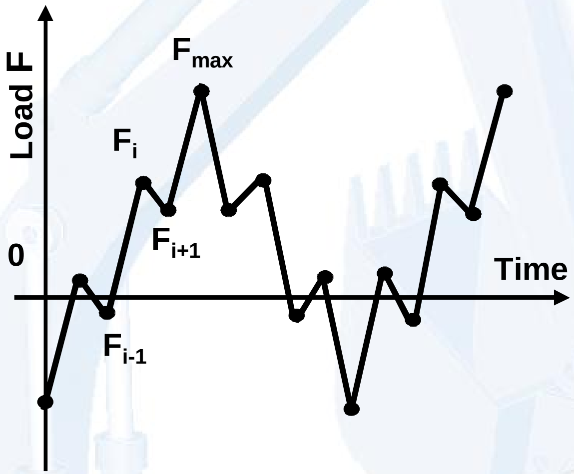
$$\sigma_{peak,i} = f\left(F_i\right); \quad f\left(F\right) = ?$$

$$\sigma_{n,i} = g\left(F_i\right); \quad g\left(F\right) = ?$$

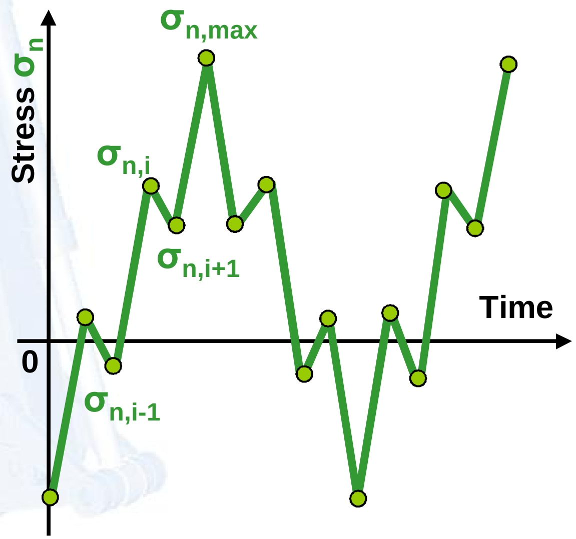


Loading and stress histories

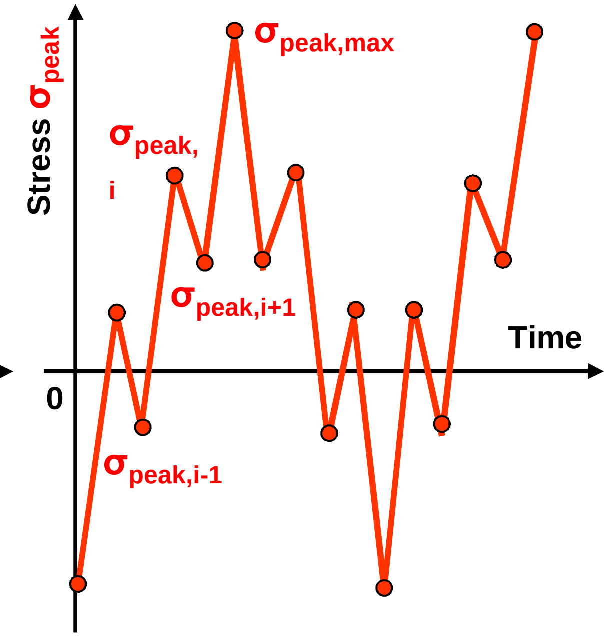
Load history, F

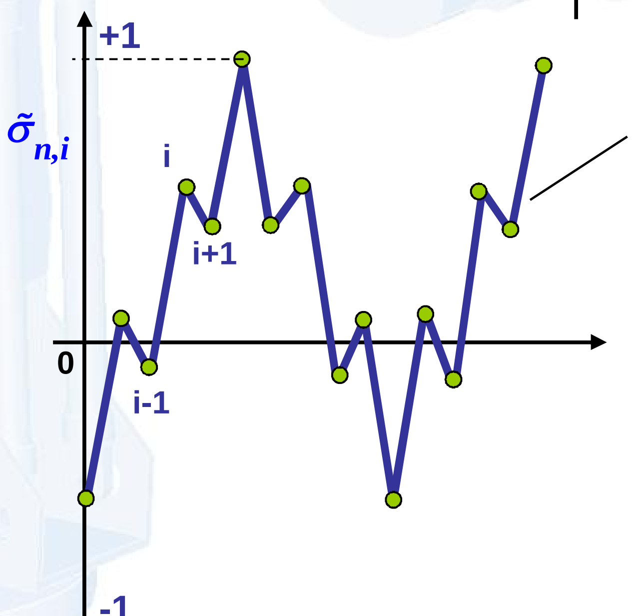


Nominal stress history, σ_n



Notch tip stress history, σ_{peak}





Characteristic non-dimensional load/stress history

$$\sigma_{n,i} = k_F F_i$$

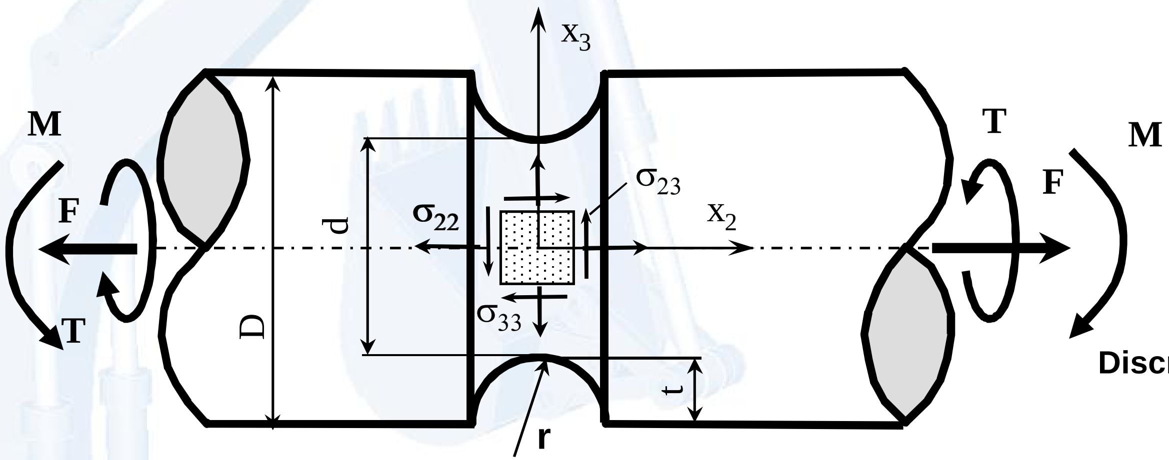
$$\sigma_{peak,i} = K_t \sigma_{n,i} = K_t k_F F_i$$

$$\tilde{\sigma}_{n,i} = \frac{F_i}{F_{\max}} = \frac{\sigma_{n,i}}{\sigma_{n,\max}} = \frac{\sigma_{peak,i}}{\sigma_{peak,\max}}$$

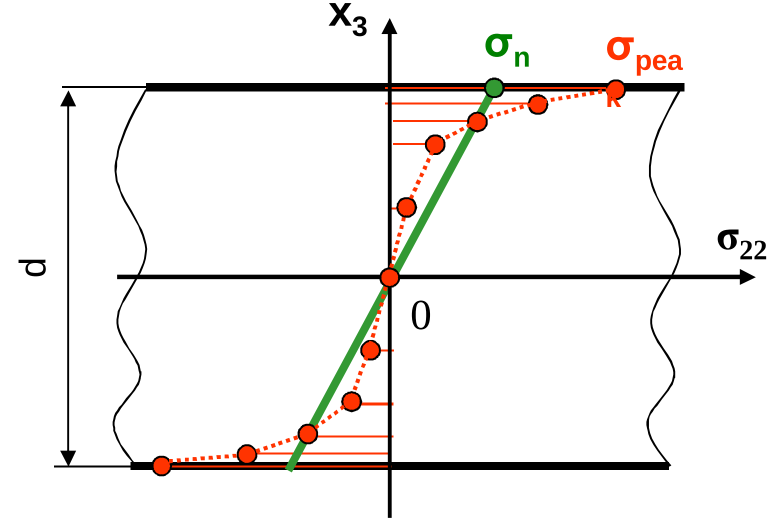
$$-1 \le \tilde{\sigma}_{n,i} \le +1$$

How to get the nominal stress σ_n from the Finite Element method stress data?

Notched shaft under axial, bending and torsion load



Discrete cross section stress distribution obtained from the FE analysis

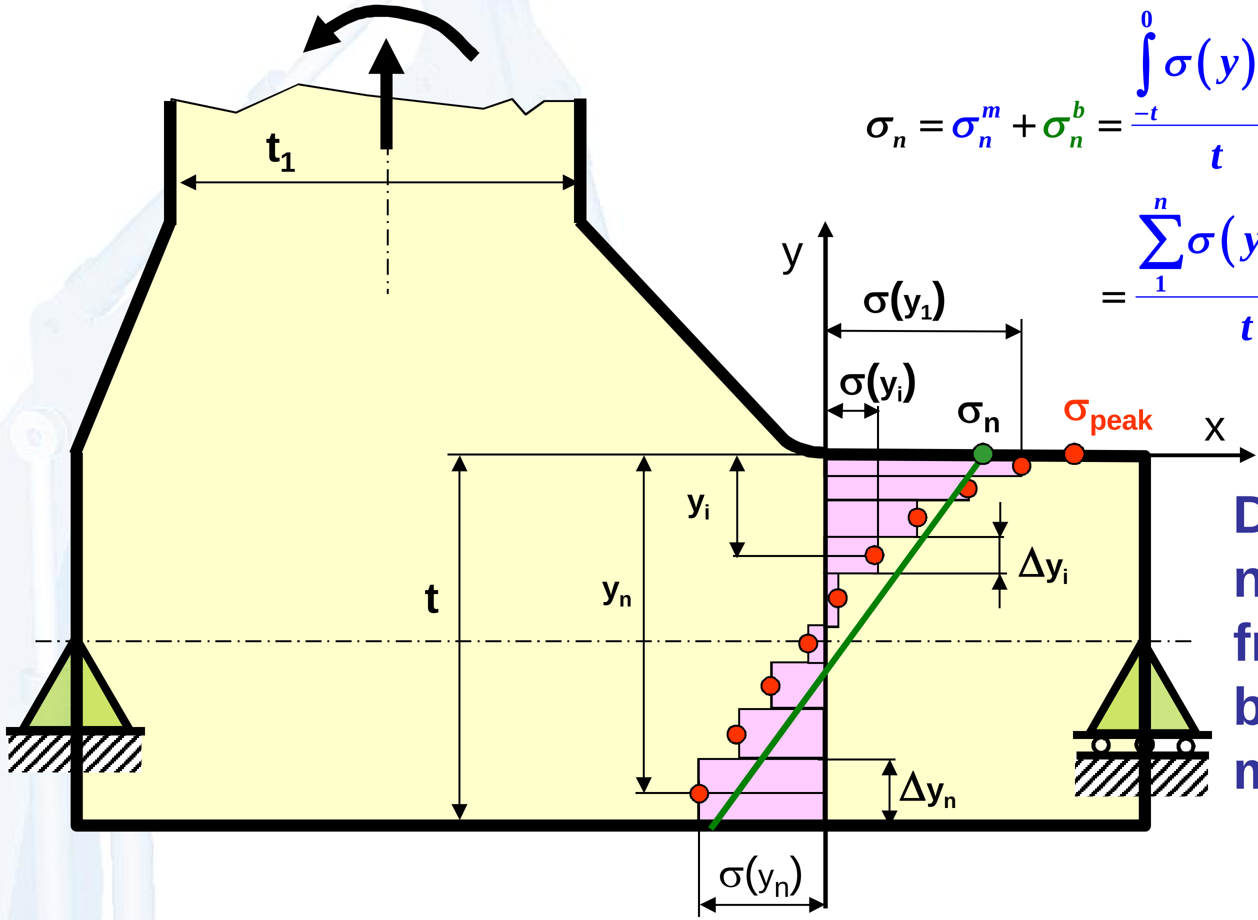


a) Run each load case separately for an unit load

b) Linearize the FE stress field for each load case

$$\sigma_n = \sigma_n^m + \sigma_n^b = \frac{\int_{-t}^{0} \sigma(y)\,dy}{t} + \frac{6 \cdot \int_{-t}^{0} \sigma(y)\,y\,dy}{t^2}$$

$$= \frac{\sum_1^n \sigma(y) \cdot \Delta y_i}{t} + \frac{6 \cdot \sum_1^n \sigma(y) \cdot y_i \cdot \Delta y_i}{t^2}$$

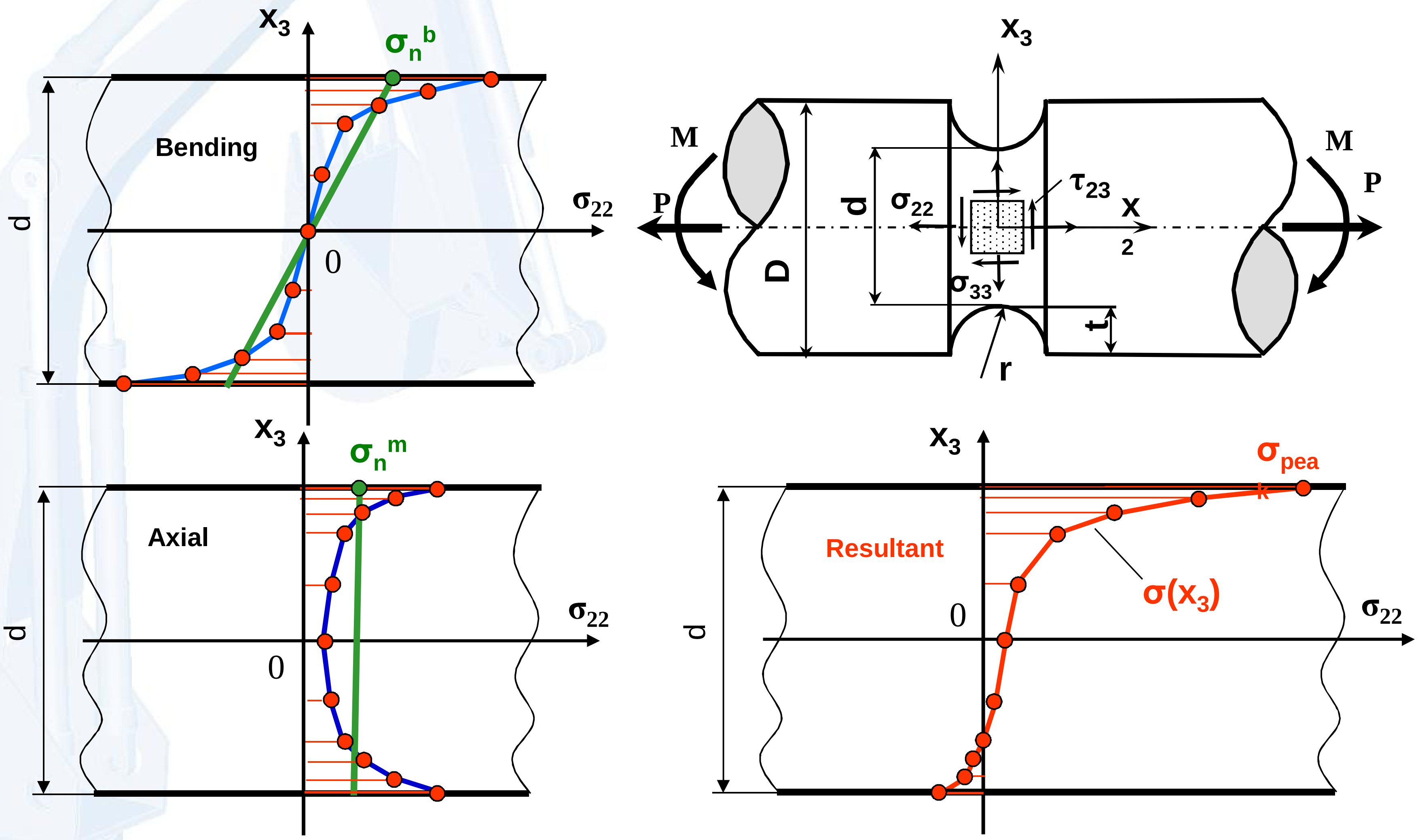


Determination of nominal stresses from discrete FE data by the linearization method

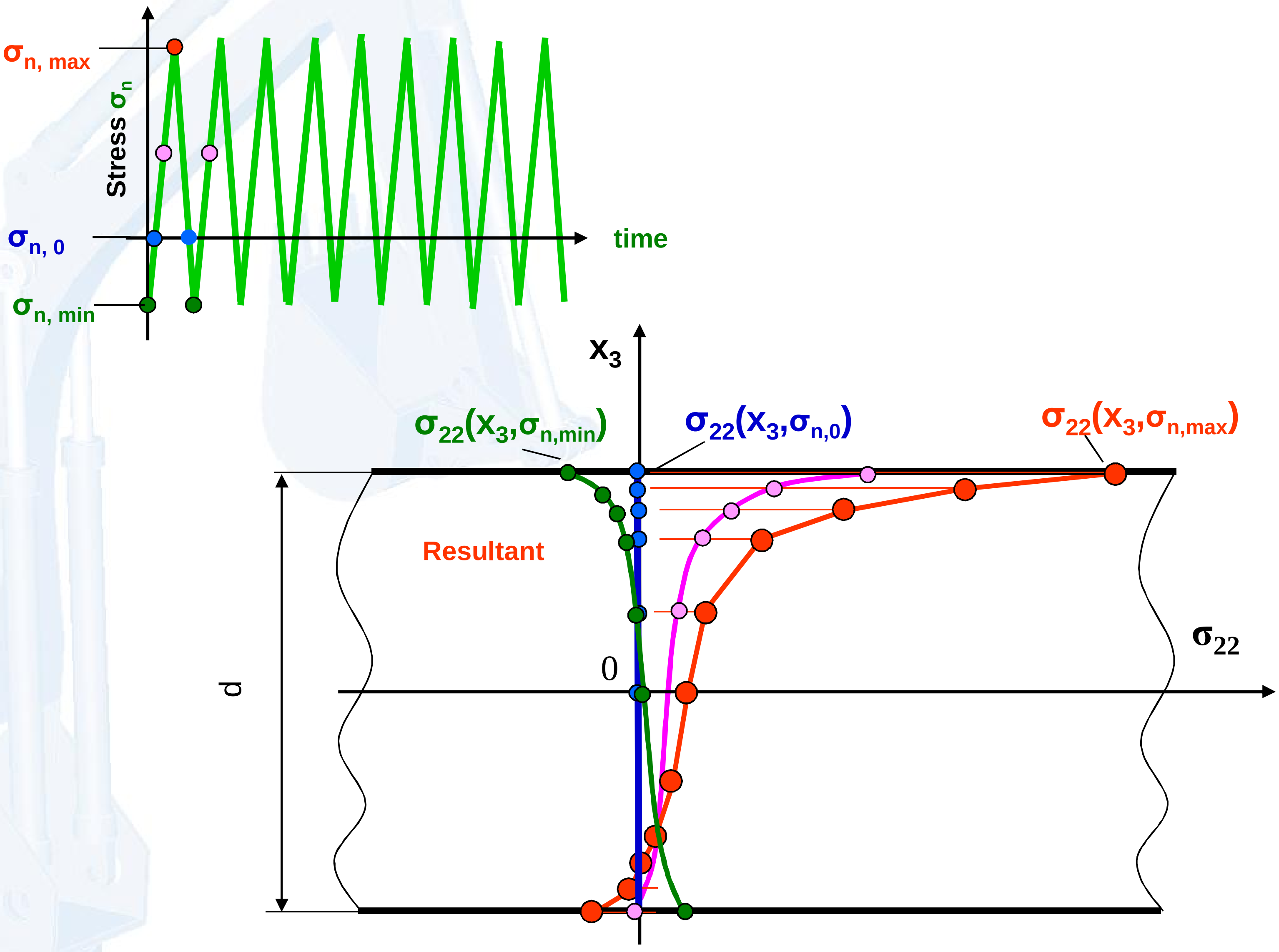
$$\sigma_n^m = \frac{P}{1 \cdot t} = \frac{\int_{-t}^{0} \sigma(y) \cdot 1 \cdot dy}{1 \cdot t} = \frac{\int_{-t}^{0} \sigma(y)\,dy}{t} = \frac{\sum_1^n \sigma(y) \cdot \Delta y_i}{t};$$

$$\sigma_n^b = \frac{c \cdot M}{I} = \frac{\frac{t}{2} \cdot \int_{-t}^{0} \sigma(y) \cdot 1 \cdot y \cdot dy}{\frac{1 \cdot t^3}{12}} = \frac{6 \int_{-t}^{0} \sigma(y) \cdot y \cdot dy}{t^2} = \frac{6 \cdot \sum_1^n \sigma(y) \cdot y_i \cdot \Delta y_i}{t^2};$$

How to get the resultant stress distribution from the Finite Element stress data? (Notched shaft under axial, bending load)

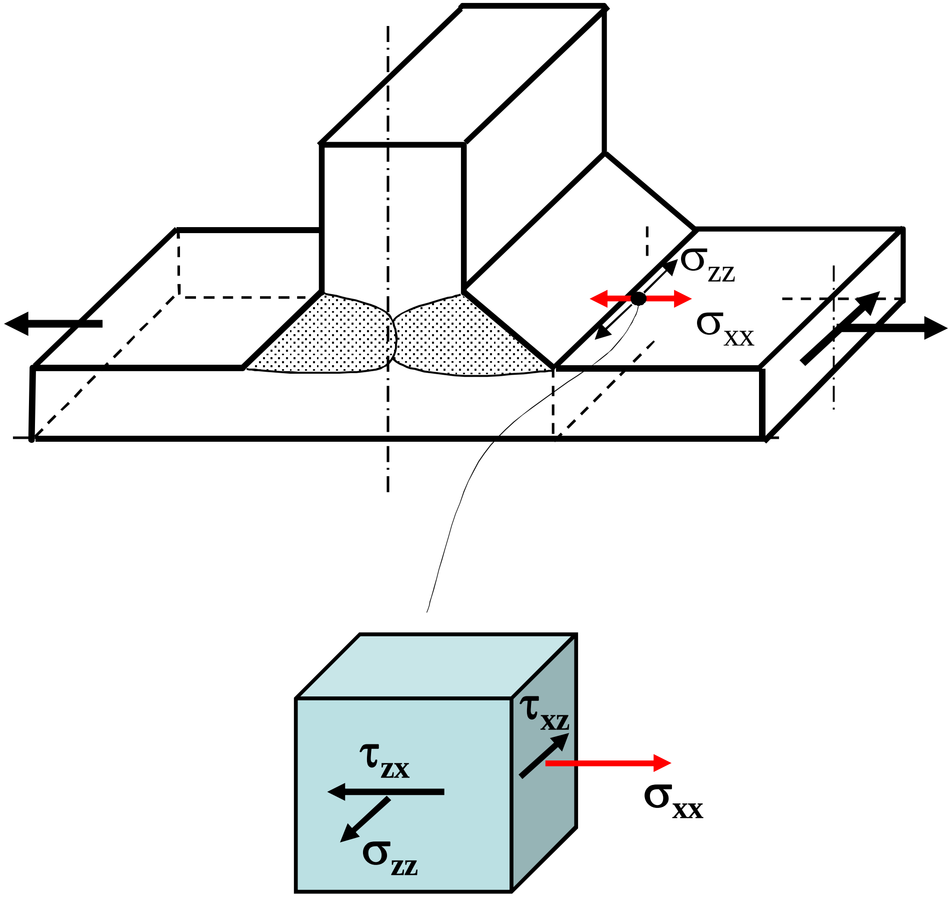


Cyclic nominal stress and corresponding fluctuating stress distribution

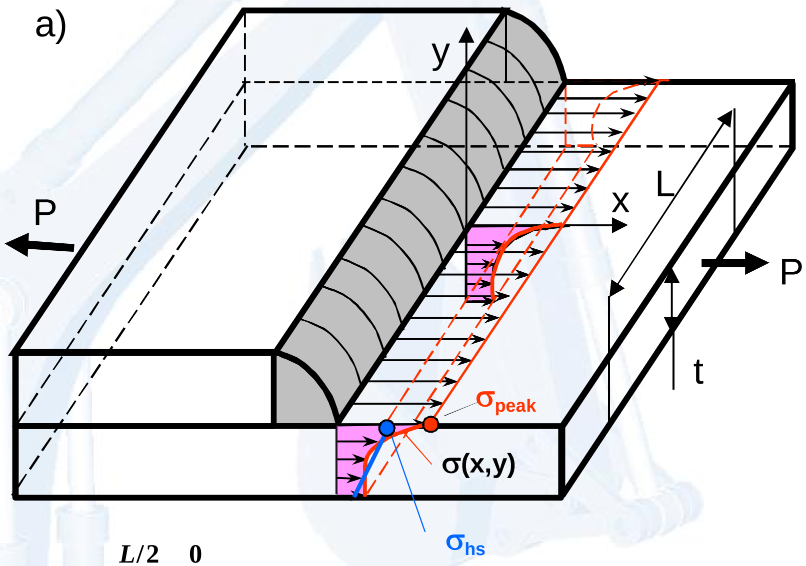


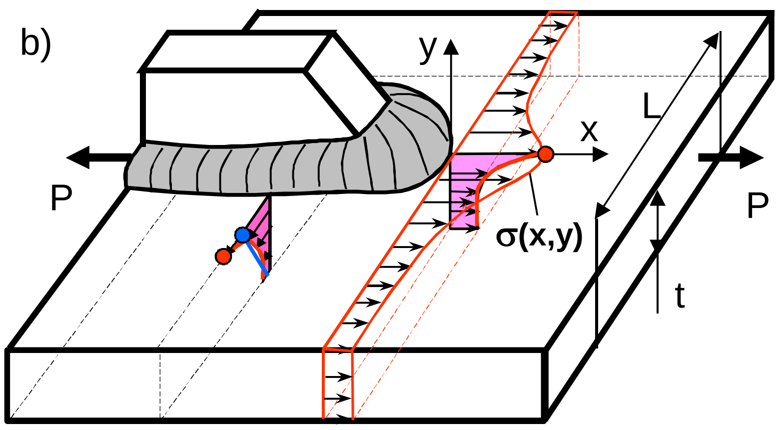
The stress state at the weld toe

- **Multiaxial state of stress at weld toe**
- **One shear and two normal stresses**
- **Due to stress concentration, σ_{xx} is the largest component**
  - ***Predominantly responsible for fatigue damage***



Determination of the nominal, σ_n, and the hot spot stress , σ_{hs}, from 3D-FE stress analysis data





$$\sigma_n = \frac{\int_{-L/2}^{L/2}\int_{-t}^{0}\sigma(x,y)\,dx\,dy}{t\cdot L} = \frac{P}{t\cdot L}$$

- depends on L and is constant along the weld toe line

$$\sigma_{hs} = \sigma_{hs}^{m} + \sigma_{hs}^{b} = \frac{\int_{-t}^{0}\sigma(x=0,y)\,dy}{t} + \frac{6\cdot\int_{-t}^{0}\sigma(x=0,y)\,y\,dy}{t^2}$$

Independent of L but it changes along the weld toe line

a) Stress distribution in the critical cross section near the cover plate ending and the nominal or the hot spot stress σ_n **(independent of length L)** and σ_{hs} **(independent of length L),**

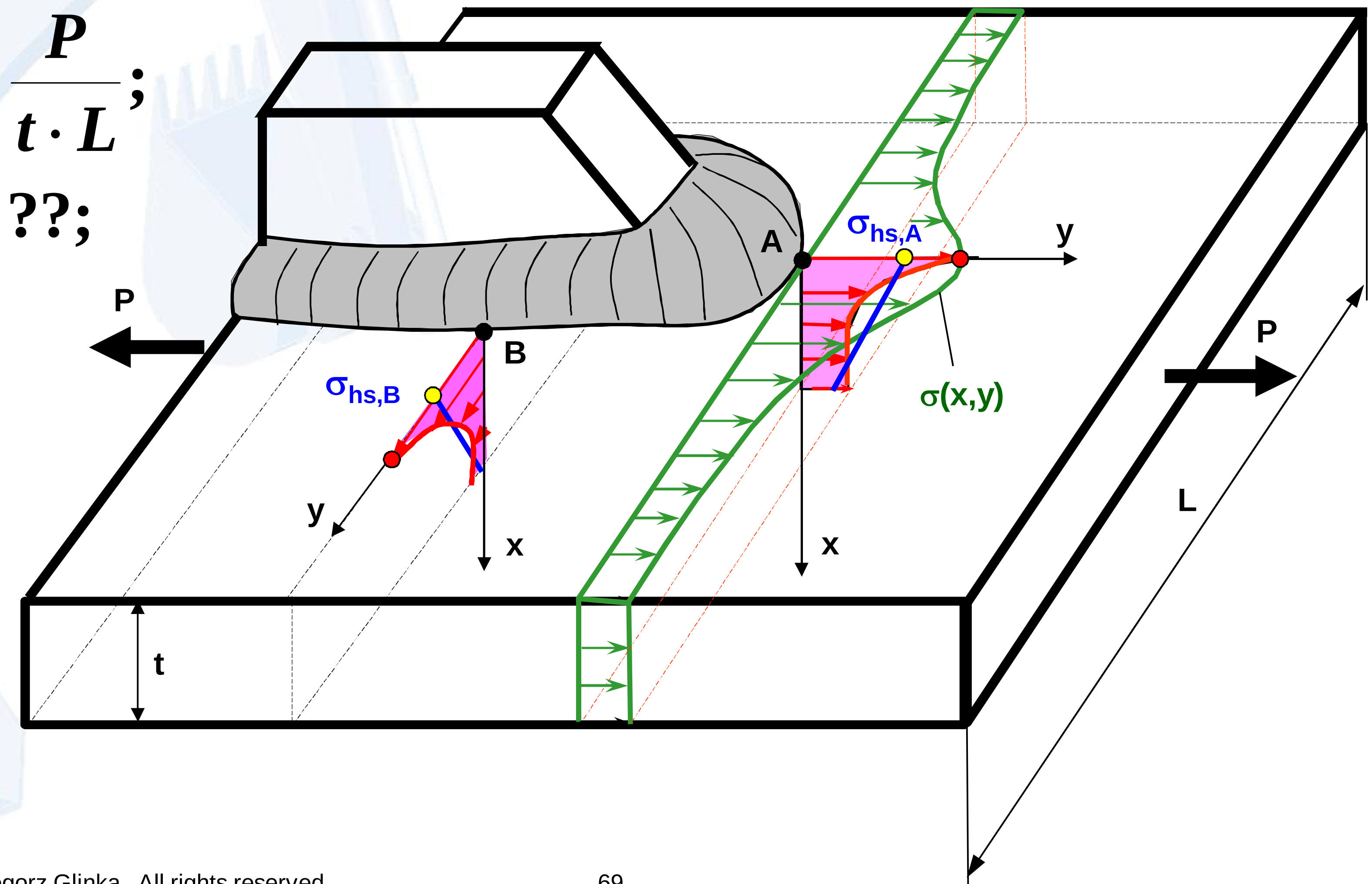
b) Stress distribution in the critical plane near the ending of a vertical attachment (gusset) and the nominal or the hot spot stress σ_n **(dependent on length L)** or σ_{hs} **(independent of length L)**

The Nominal Stress σ_n versus the local Hot Spot Stress σ_{hs}

$$\sigma_{hs,A} = \sigma^{m}_{hs,A} + \sigma^{b}_{hs,A}; \quad \sigma_{hs,B} = \sigma^{m}_{hs,B} + \sigma^{b}_{hs,B};$$

$$\sigma_{n,A} = \frac{P}{t \cdot L};$$

$$\sigma_{n,B} = ??;$$



Example:

Preparation and Analysis of Representative Stress/Load History:

The Rainflow Cycle Counting Procedure

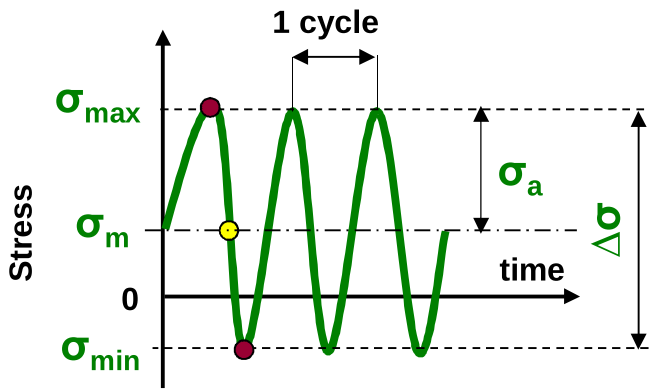
Stress/Load Analysis - Cycle Counting Procedure and Presentation of Results

The measured stress, strain, or load history is given usually in the form of a time series, i.e. a sequence of discrete values of the quantity measured in equal time intervals. When plotted in the stress-time space the discrete point values can be connected resulting in a continuously changing signal. However, the time effect on the fatigue performance of metals (except aggressive environments) is negligible in most cases. Therefore the excursions of the signal, represented by amplitudes or ranges, are the most important quantities in fatigue analyses. Subsequently, the knowledge of the reversal point values, denoted with large diamond symbols in the next Figure, is sufficient for fatigue life calculations. For that reason the intermediate values between subsequent reversal points can be deleted before any further analysis of the loading/stress signal is carried out. An example of a signal represented by the reversal points only is shown in slide no. 141. The fatigue damage analysis requires decomposing the signal into elementary events called 'cycles'. Definition of a loading/stress cycle is easy and unique in the case of a constant amplitude signal as that one shown in the figures. A stress/loading cycle, as marked with the thick line, is defined as an excursion starting at one point and ending at the next subsequent point having exactly the same magnitude and the same sign of the second derivative. The maximum, minimum, amplitude or range and mean stress values characterise the cycle.

$$\Delta\sigma = \left|\sigma_{\max} - \sigma_{\min}\right| \; - \; \textit{stress range}$$

$$\sigma_a = \frac{\left|\sigma_{\max} - \sigma_{\min}\right|}{2} \; - \; \textit{stress amplitude}$$

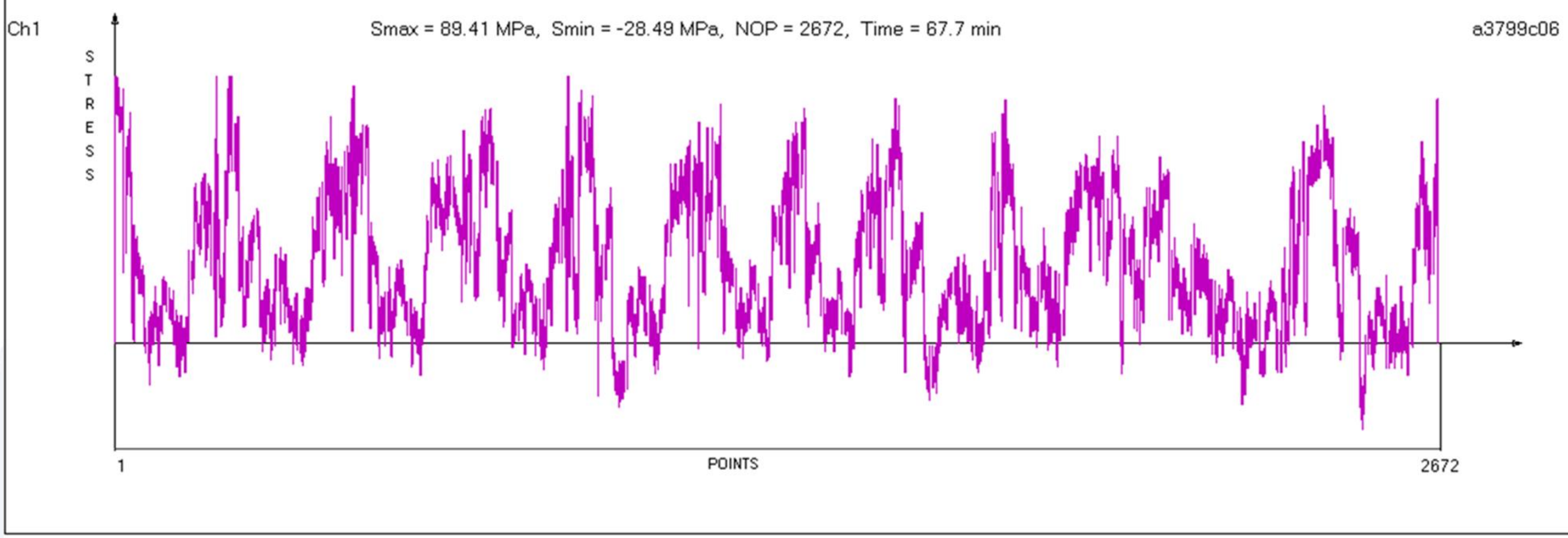
$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} \; - \; \textit{mean stress}$$



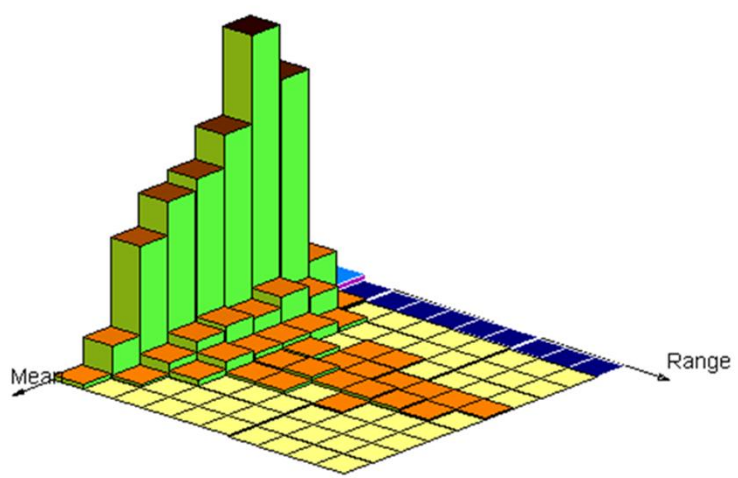
Unfortunately, the cycle definition is not simple in the case of a variable amplitude signal. The only non-dubious quantity, which can easily be defined, is a reversal, example of which is marked with the thick line in the Figures below.

Removing material from a clay mine in Tennessee

Stress History



RNF Matrix

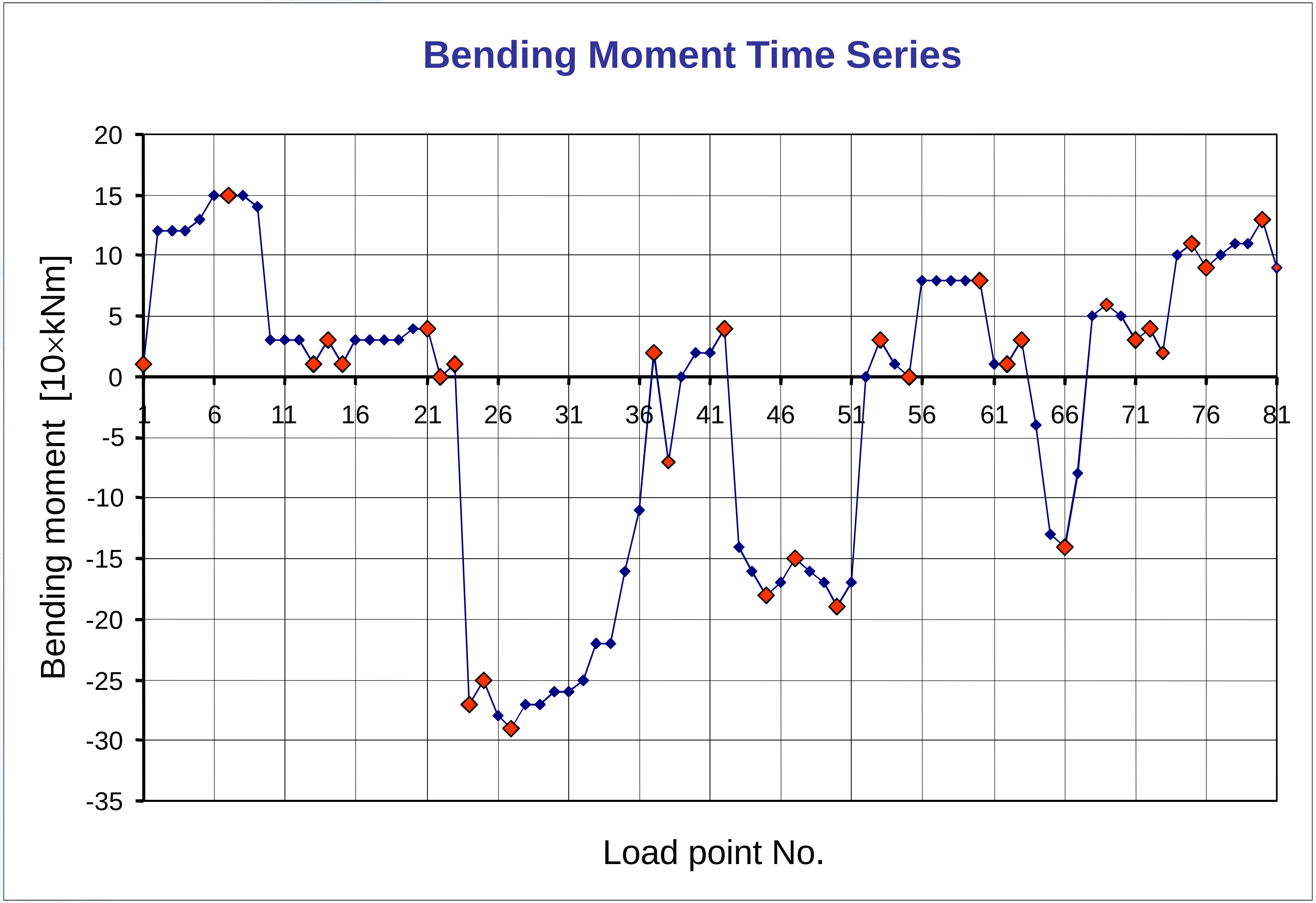


RNF Matrix Information

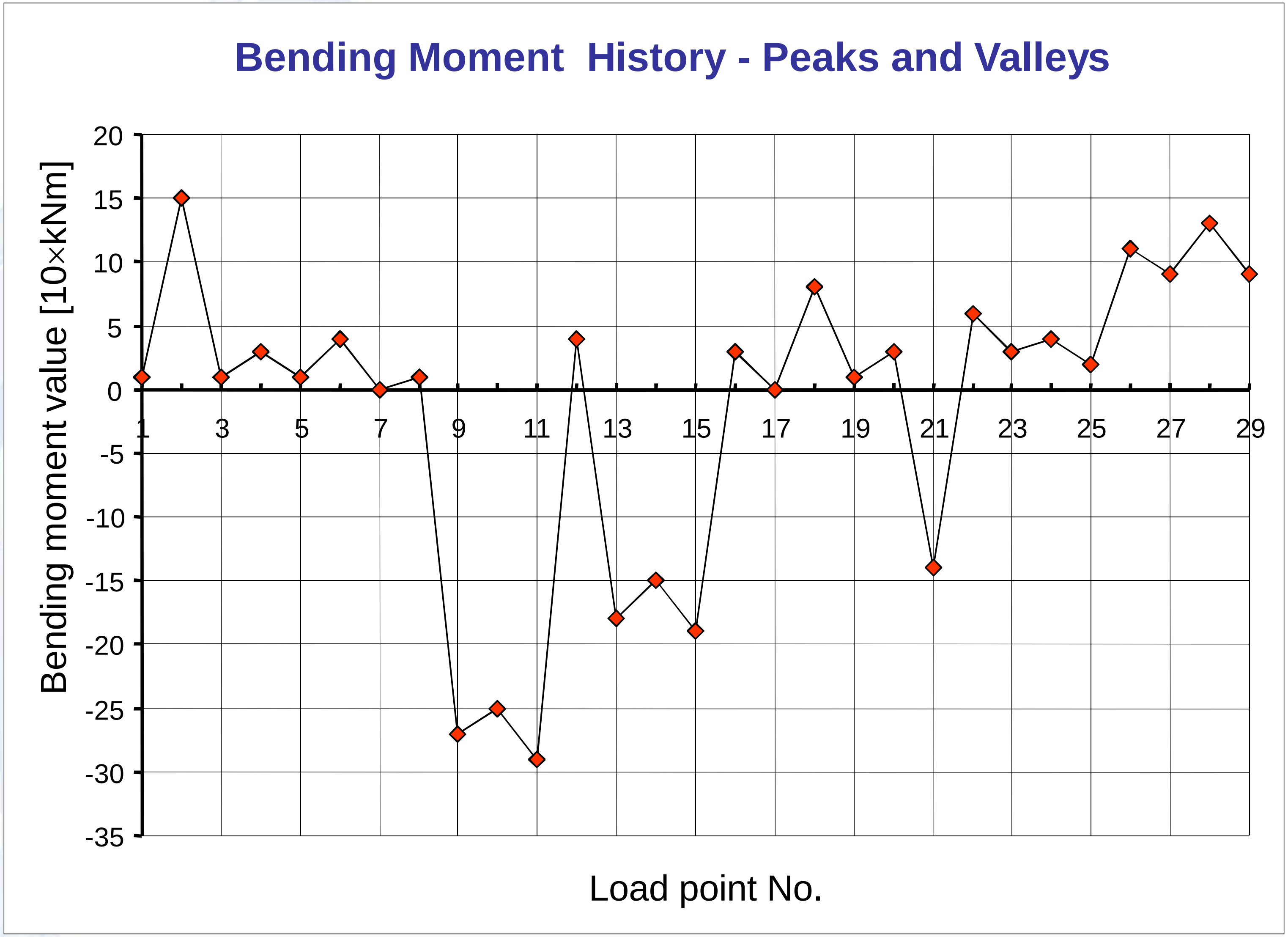
Mean Stress : Sm(1) = -21.59 MPa, Mean Stress Levels: 10

Level	Range [MPa]	Cycles
1	117.7	0
2	105.93	0
3	94.16	0
4	82.39	0
5	70.62	0
6	58.85	0
7	47.08	0

Stress Range Levels: 10 Omitted cycles: 1



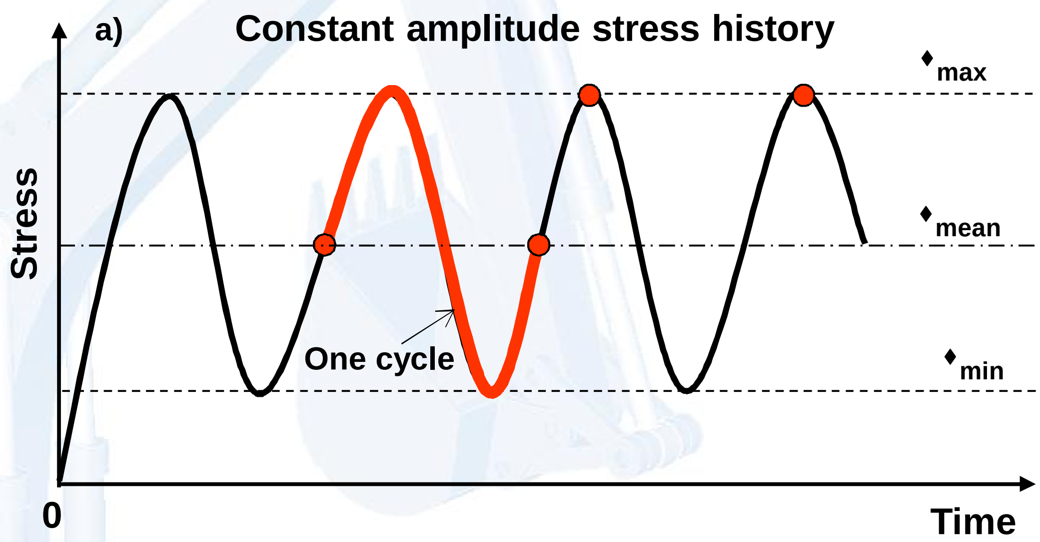
Bending Moment measurements obtained at constant time intervals

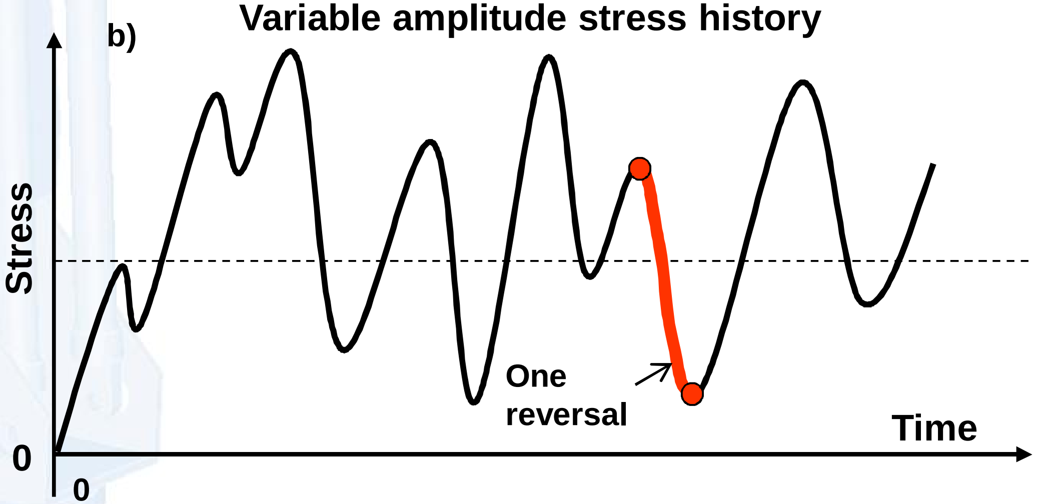


Bending Moment signal represented by the reversal point values

Constant and Variable Amplitude Stress Histories;

Definition of the Stress Cycle & Stress Reversal





$$\Delta\sigma = \sigma_{max} - \sigma_{min};$$

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2};$$

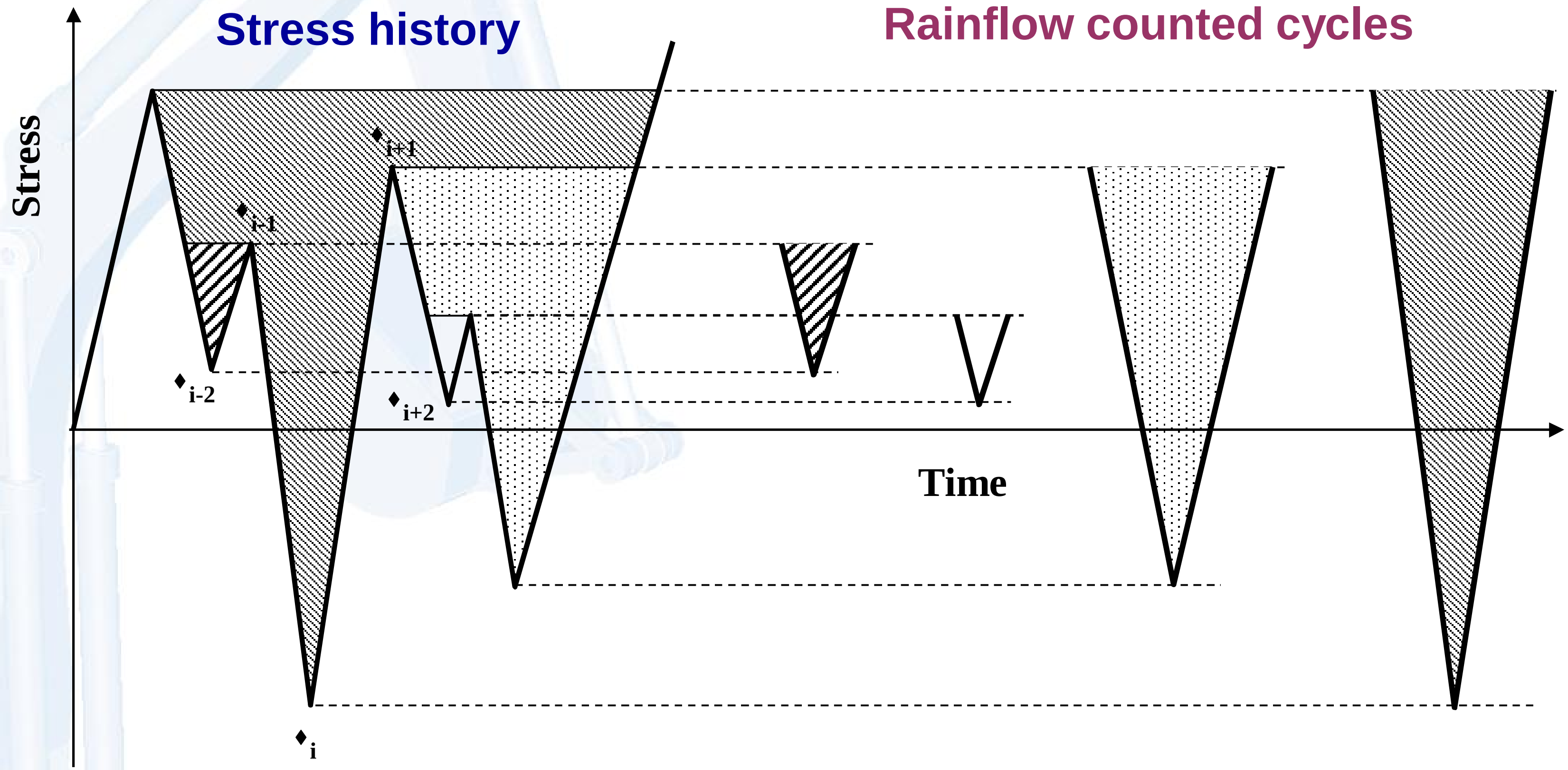
$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Stress Reversals and Stress Cycles in a Variable Amplitude Stress History

The *reversal* is simply an excursion between two-consecutive reversal points, i.e. an excursion between subsequent peak and valley or valley and peak.

In recent years the *rainflow* cycle counting method has been accepted world-wide as the most appropriate for extracting stress/load cycles for fatigue analyses. The *rainflow* cycle is defined as a stress excursion, which when applied to a deformable material, will generate a closed *stress-strain hysteresis loop*. It is believed that the surface area of the stress-strain hysteresis loop represents the amount of damage induced by given cycle. An example of a short stress history and its *rainflow counted cycles* content is shown in the following Figure.

Stress History and the "Rainflow" Counted Cycles



A *rainflow counted cycle* is identified when any two adjacent reversals in the stress history satisfy the following relation:

$$ABS\left|\sigma_{i-1} - \sigma_i\right| \leq ABS\left|\sigma_i - \sigma_{i+1}\right|$$

The Mathematics of the Cycle Rainflow Counting Method for Fatigue Analysis of Fluctuating Stress/Load Histories

A ***rainflow counted cycle*** is identified when any two adjacent reversals in thee stress history satisfy the following relation:

$$ABS\left|\sigma_{i-1}-\sigma_{i}\right| \leq ABS\left|\sigma_{i}-\sigma_{i+1}\right|$$

The stress amplitude of such a cycle is:

$$\sigma_{a}=\frac{ABS\left|\sigma_{i-1}-\sigma_{i}\right|}{2}$$

The stress range of such a cycle is:

$$\Delta\sigma = ABS\left|\sigma_{i-1}-\sigma_{i}\right|$$

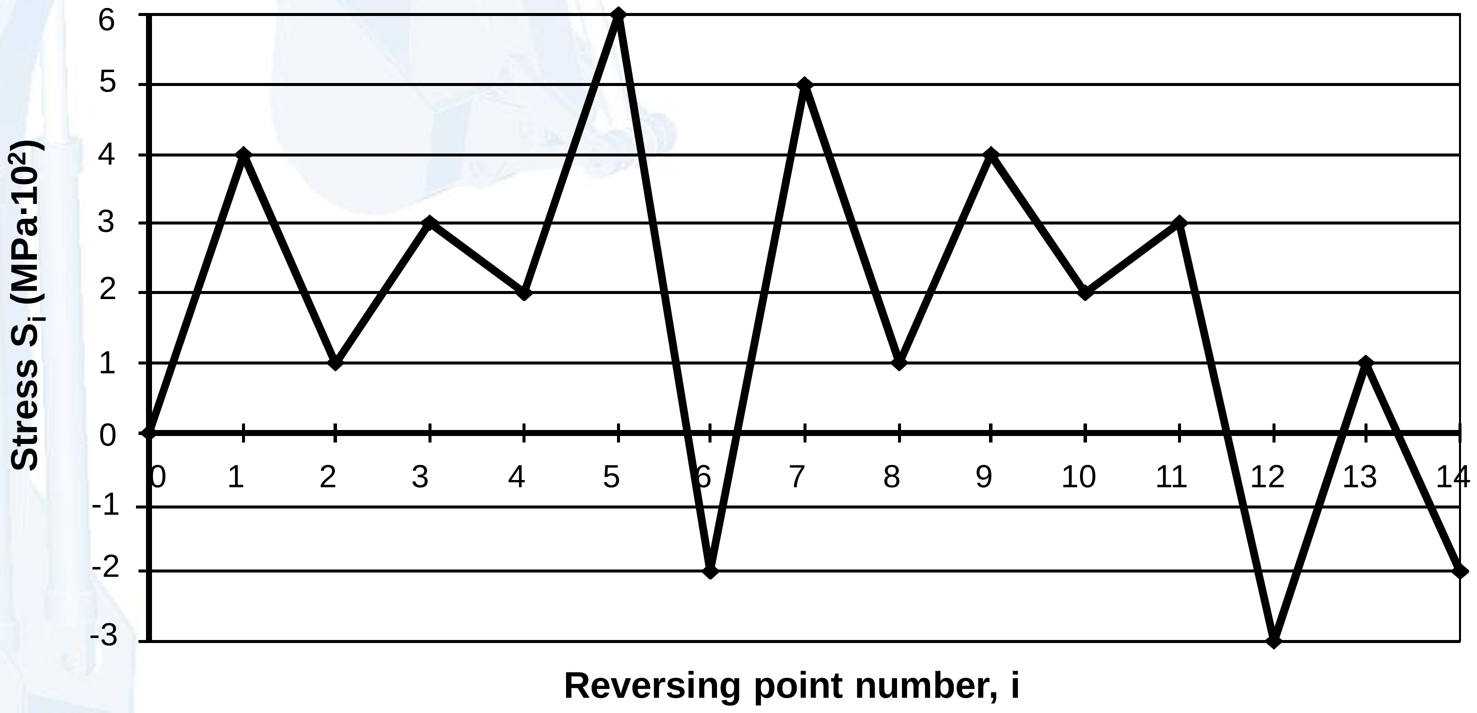
The mean stress of such a cycle is:

$$\sigma_{m}=\frac{\sigma_{i-1}+\sigma_{i}}{2}$$

The rainflow cycle counting procedure - example

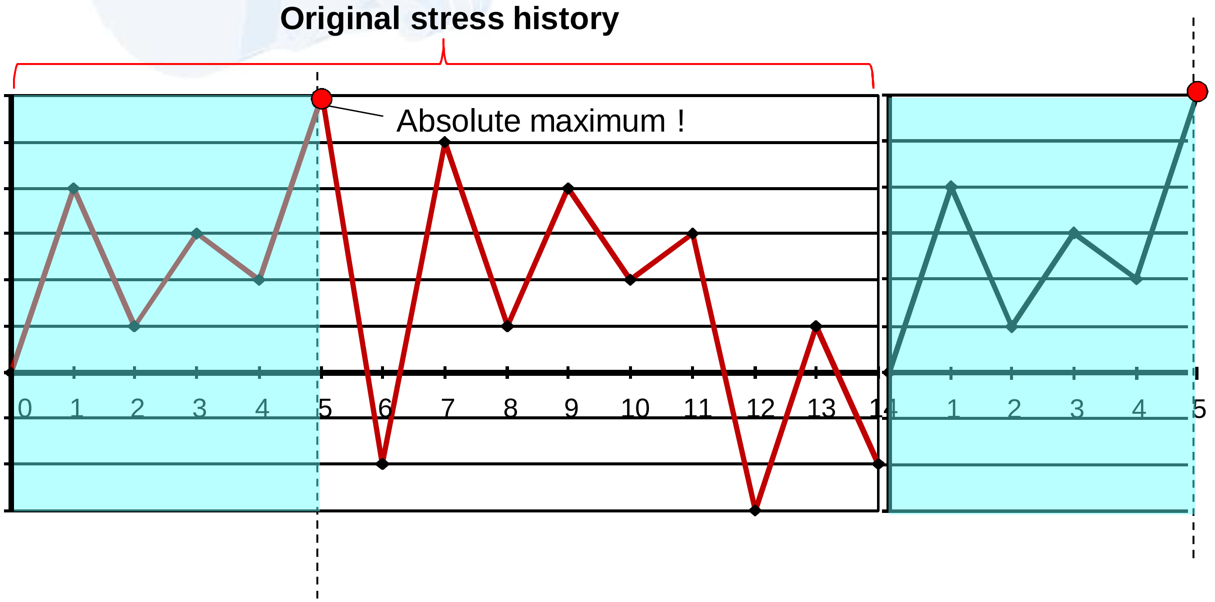
Determine stress ranges, ΔS_i, and corresponding mean stresses, S_{mi} for the stress history given below. Use the 'rainflow' counting procedure.

S_i= 0, 4, 1, 3, 2, 6, -2, 5, 1, 4, 2, 3, -3, 1, -2 (units: MPa·10²)

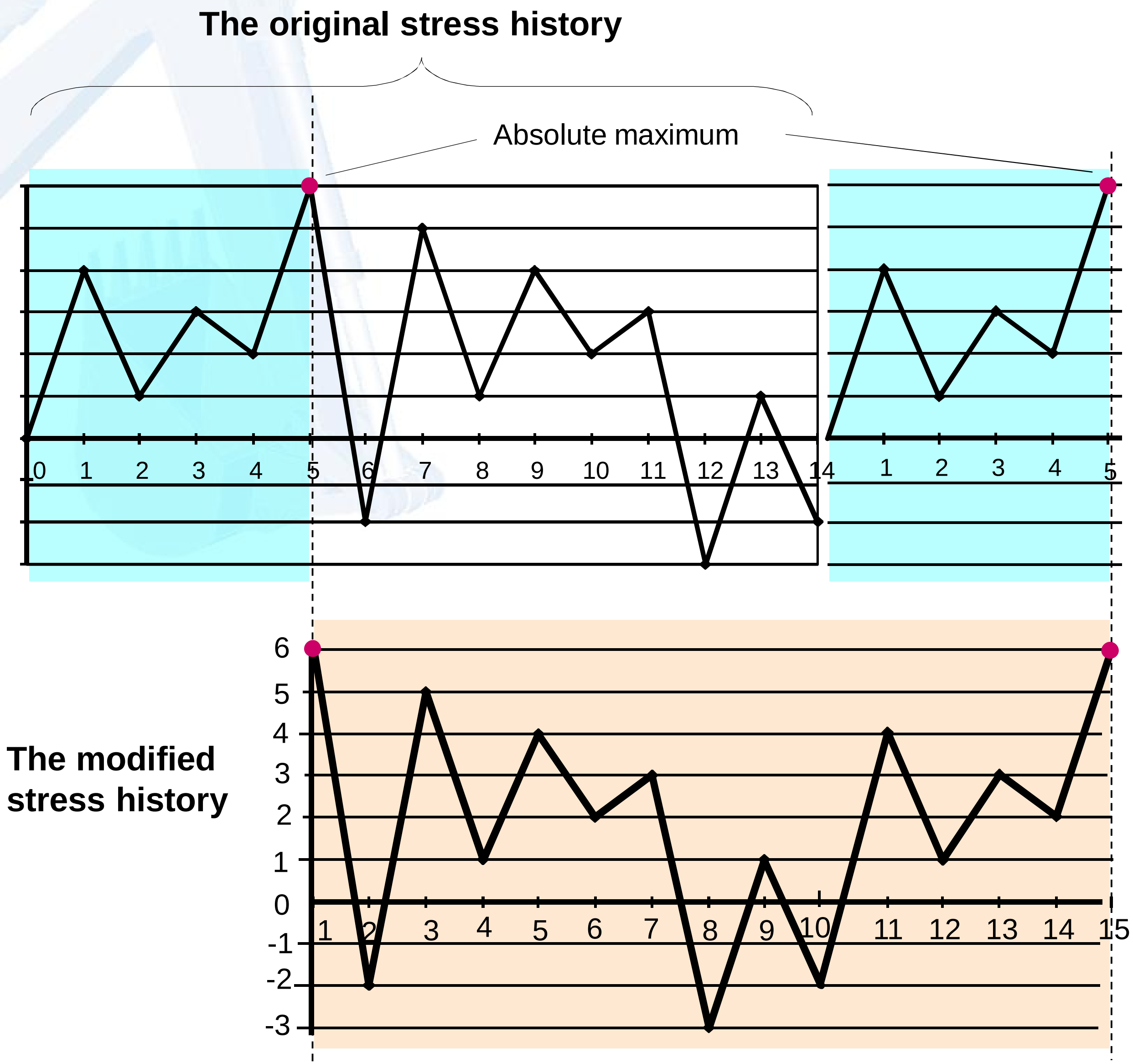


The ASTM rainflow counting procedure

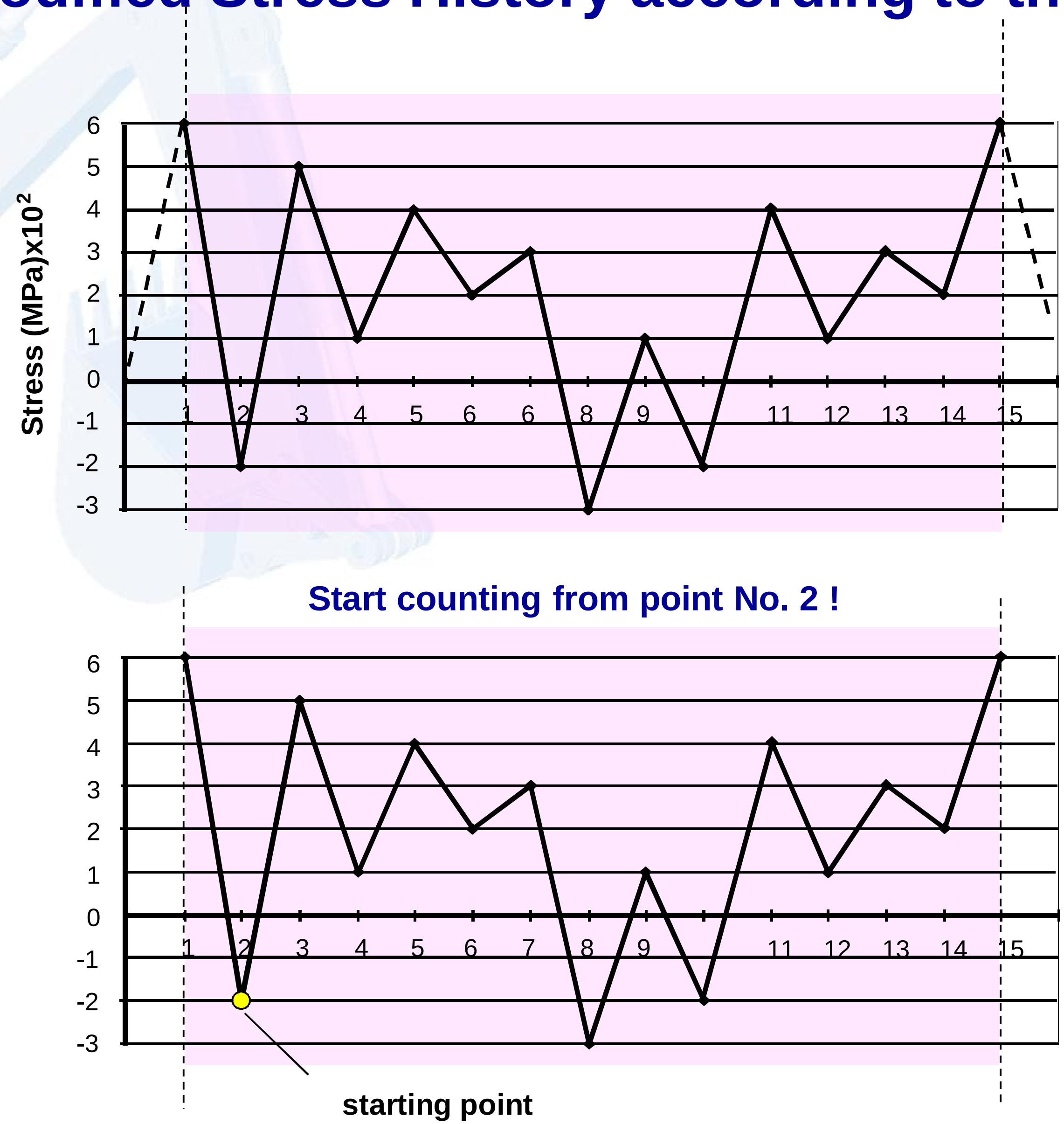
1. Find the reversing point with highest absolute stress magnitude,
2. The part of the stress history before the maximum absolute attach to the end of the history,
3. Perform the rainflow counting on the re-arranged stress history, i.e. from maximum to maximum



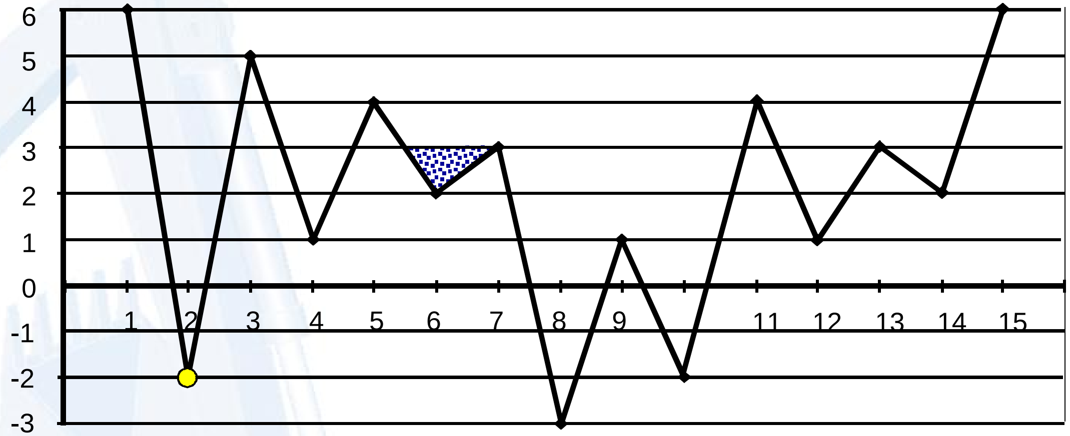
The ASTM modification of the Stress History



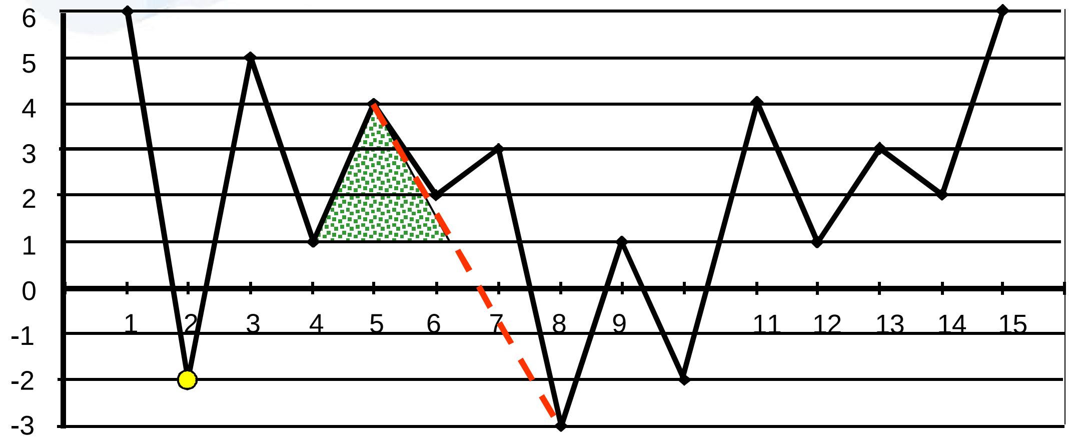
The Modified Stress History according to the ASTM



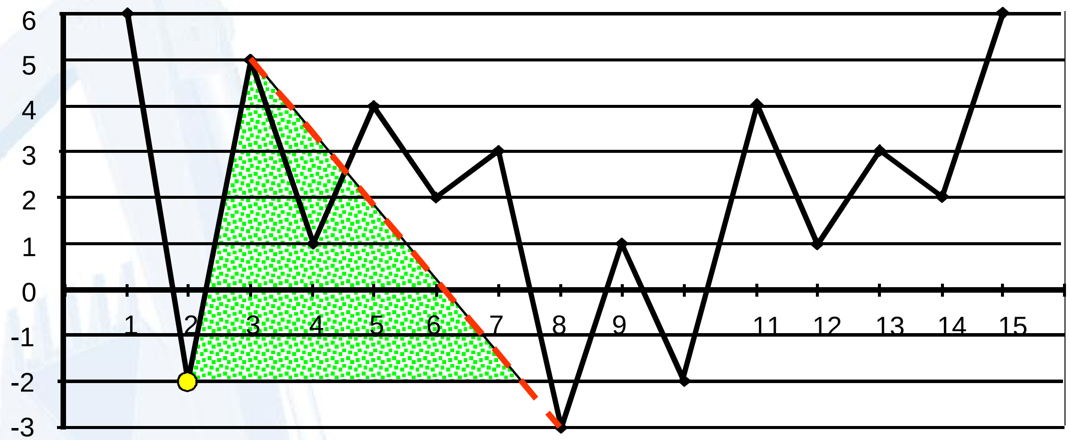
Start counting from the point No. 2 !!



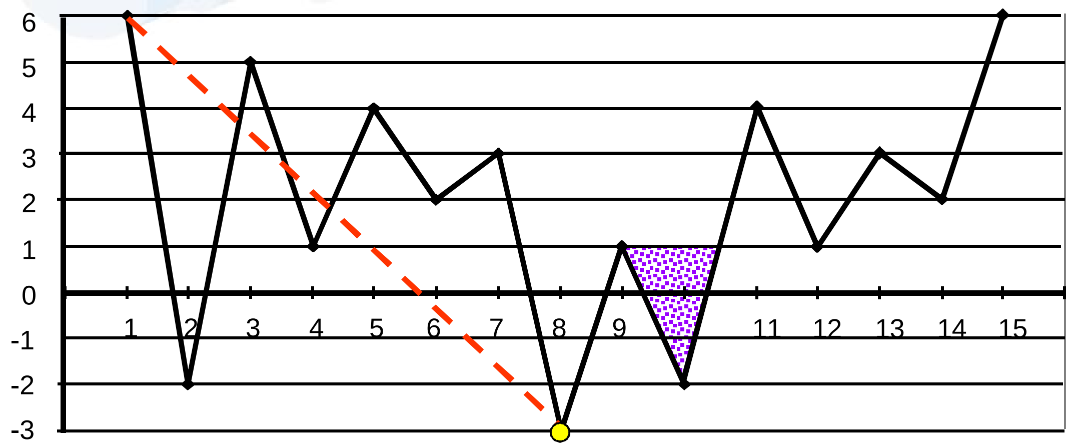
$$\Delta\sigma_{6-7} = |\sigma_6 - \sigma_7| = |3 - 2| = 1; \qquad \sigma_{m,6-7} = \frac{\sigma_6 + \sigma_7}{2} = \frac{3+2}{2} = 2.5;$$



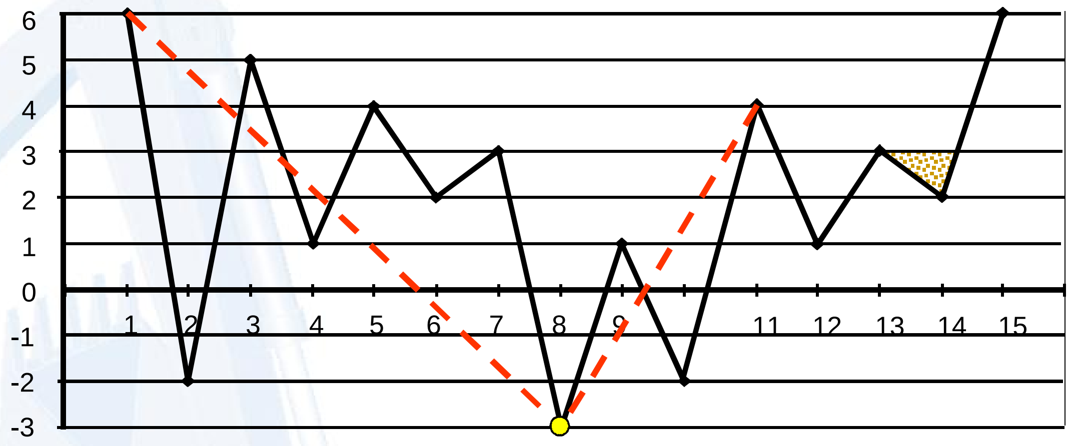
$$\Delta\sigma_{4-5} = |\sigma_4 - \sigma_5| = |1 - 4| = 3; \qquad \sigma_{m,4-5} = \frac{\sigma_4 + \sigma_5}{2} = \frac{1+4}{2} = 2.5;$$



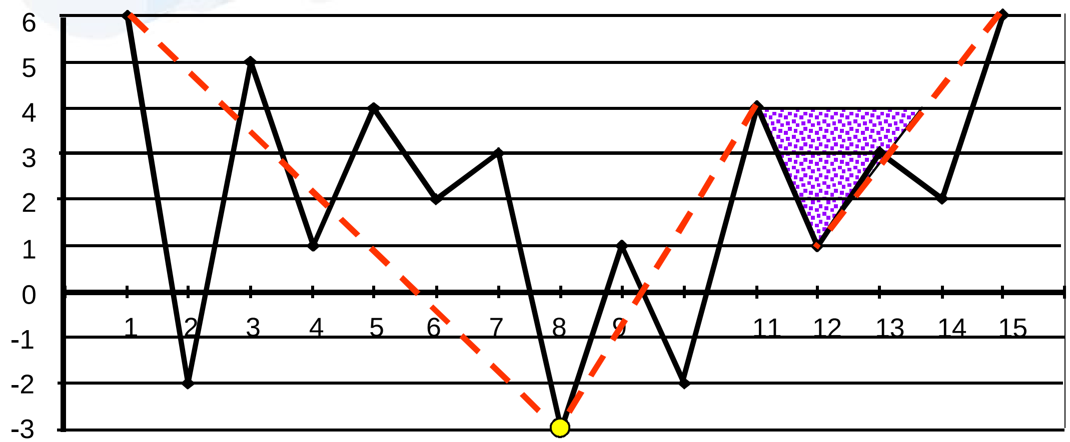
$$\Delta\sigma_{2-3} = \left|\sigma_2 - \sigma_3\right| = \left|-2-(5)\right| = 7; \qquad \sigma_{m,6-7} = \frac{\sigma_2+\sigma_3}{2} = \frac{-2+5}{2} = 1.5;$$



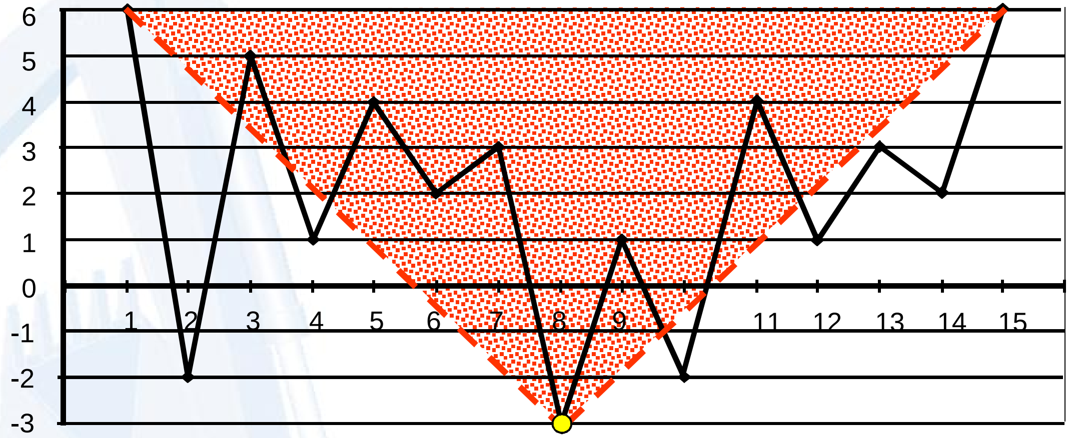
$$\Delta\sigma_{9-10} = \left|\sigma_9 - \sigma_{10}\right| = \left|1-(-2)\right| = 3; \qquad \sigma_{m,9-10} = \frac{\sigma_9+\sigma_{10}}{2} = \frac{1-2}{2} = -0.5;$$



$$\Delta\sigma_{13-14} = \left|\sigma_{13} - \sigma_{14}\right| = \left|3-2\right| = 1; \qquad \sigma_{m,13-14} = \frac{\sigma_{13}+\sigma_{14}}{2} = \frac{3+2}{2} = 2.5;$$



$$\Delta\sigma_{11-12} = \left|\sigma_{11} - \sigma_{12}\right| = \left|4-1\right| = 3; \qquad \sigma_{m,11-12} = \frac{\sigma_{11}+\sigma_{12}}{2} = \frac{4+1}{2} = 2.5;$$



$$\Delta\sigma_{1-8} = \left|\sigma_1 - \sigma_8\right| = \left|6-(-3)\right| = 9; \qquad \sigma_{m,1-8} = \frac{\sigma_1 + \sigma_8}{2} = \frac{6-3}{2} = 1.5;$$

Cycles counted –ASTM method

1. $\Delta\sigma_{6-7}$ =1; $\sigma_{m,6-7}$ = 2.5;
2. $\Delta\sigma_{4-5}$ =3; $\sigma_{m,4-5}$ = 2.5;
3. $\Delta\sigma_{13-14}$=1; $\sigma_{m,10-11}$= 2.5;
4. $\Delta\sigma_{11-12}$=3; $\sigma_{m,11-12}$= 2.5;
5. $\Delta\sigma_{2-3}$ =7; $\sigma_{m,2-3}$ = 1.5;
6. $\Delta\sigma_{9-10}$ =3; $\sigma_{m,9-10}$ =-0.5;
7. $\Delta\sigma_{1-8}$ =9; $\sigma_{m,1-8}$ = 1.5;

Extracted rainflow cycles, $\Delta\sigma$- $\Delta\sigma_m$

Mean stress, σ_m

Stress range, $\Delta\sigma$

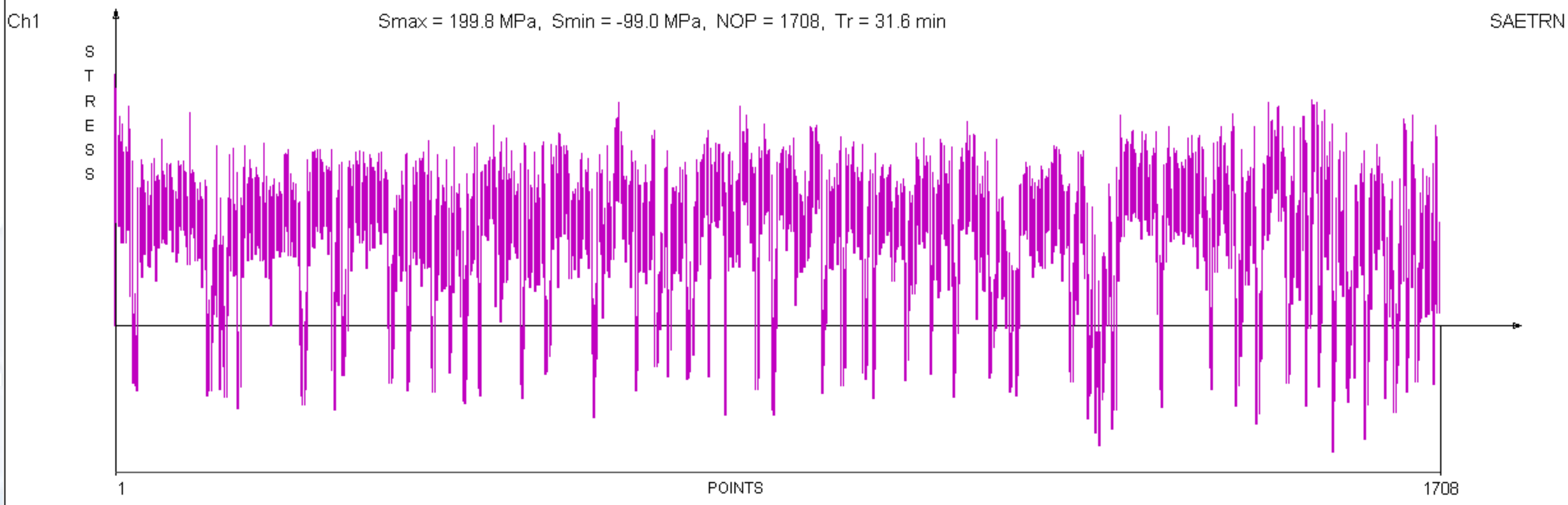
$\Delta\sigma/\sigma_m$	-32	-22	-13	-3.2	6.44	16.1	25.7	35.3	45	54	64.1	73.7	83.3	92.9	103	112	122	131	141	151	$\Delta\sigma$
298.8	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
283.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
268.9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
254	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
239	0	0	0	0	0	0	0	1	2	2	0	0	0	0	0	0	0	0	0	0	5
224.1	0	0	0	0	0	0	0	0	2	2	1	0	0	0	0	0	0	0	0	0	5
209.2	0	0	0	0	0	0	0	3	4	5	2	0	0	0	0	0	0	0	0	0	14
194.2	0	0	0	0	0	1	0	1	7	2	0	0	0	0	0	0	0	0	0	0	11
179.3	0	0	0	0	0	0	1	0	4	4	0	0	0	0	0	0	0	0	0	0	9
164.3	0	0	0	0	1	0	0	0	3	1	0	0	0	0	0	0	0	0	0	0	5
149.4	0	0	0	0	0	0	1	0	0	0	0	2	1	0	0	0	0	0	0	0	4
134.5	0	0	0	0	0	0	0	0	0	0	0	4	1	1	0	0	0	0	0	0	6
119.5	0	1	1	0	0	0	0	0	0	0	3	1	5	1	2	0	0	0	0	0	14
104.6	0	0	1	2	1	0	0	0	0	2	4	3	7	3	2	1	2	1	0	0	29
89.64	0	1	2	3	7	2	0	0	0	1	2	8	10	7	5	6	2	1	0	0	57
74.7	1	1	3	4	3	5	0	1	2	2	10	18	23	20	17	11	4	1	0	0	126
59.76	2	1	5	7	4	1	4	5	1	2	11	20	34	31	31	28	9	7	1	1	205
44.82	1	6	9	7	9	7	10	3	3	8	15	37	49	64	62	41	16	11	2	1	361
29.88	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14.94	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

854

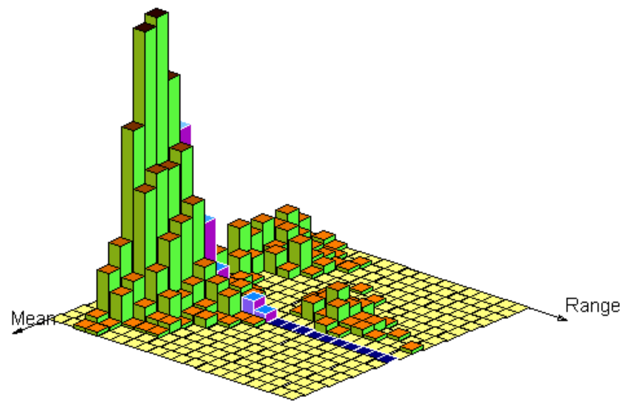
Total number of cycles, N=854

Extracted **rainflow** cycles, $\Delta\sigma$- $\Delta\sigma_m$

Stress History



RNF Matrix



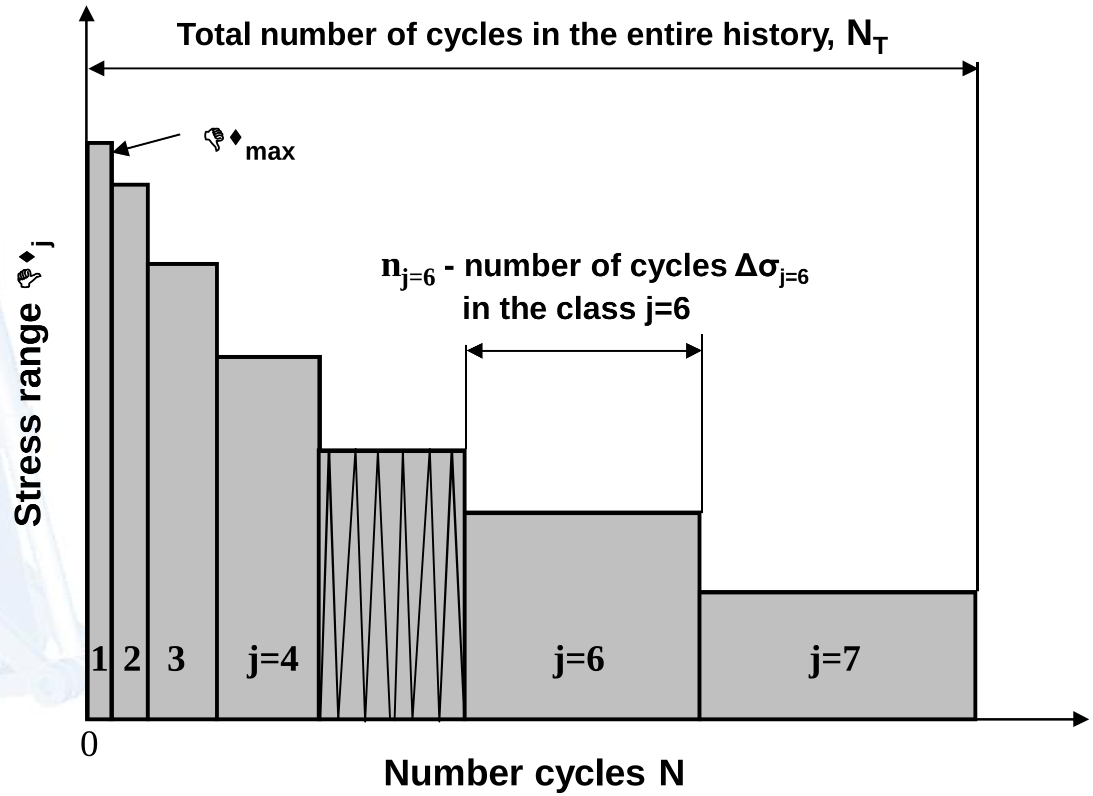
RNF Matrix Information

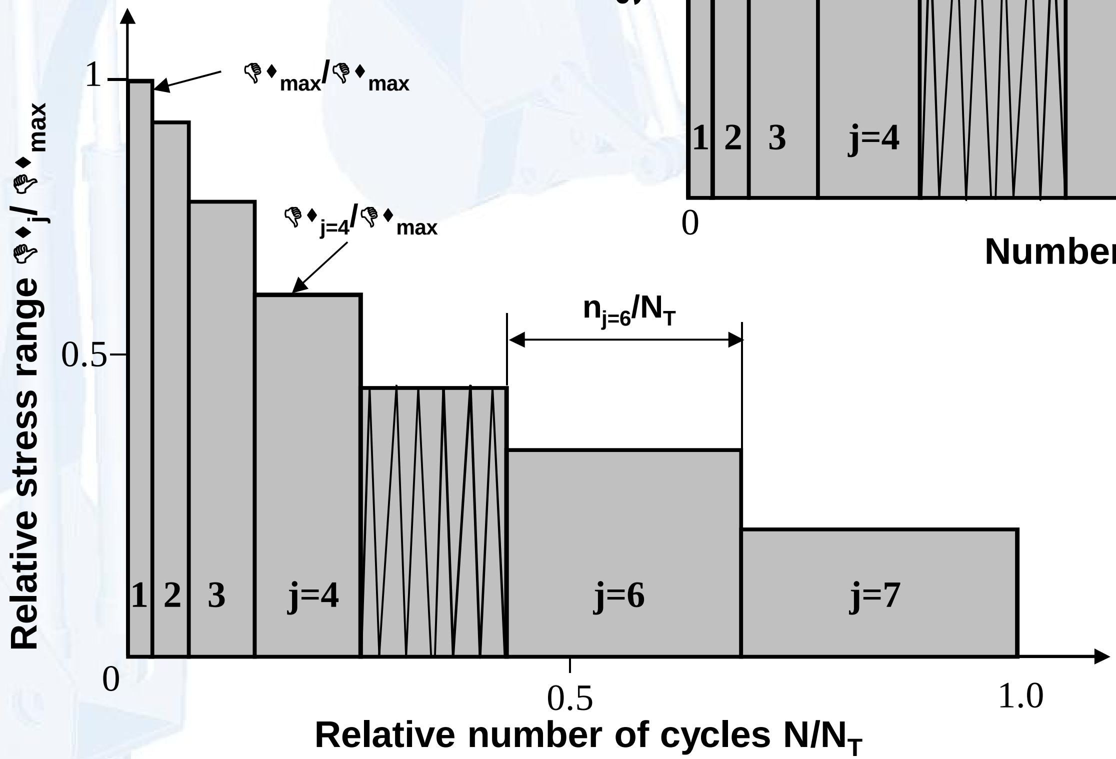
Mean Stress : Sm(12) = 73.72 MPa, Mean Stress Levels: 20

Level	Range [MPa]	Cycles
1	298.8	0
2	283.86	0
3	268.92	0
4	253.98	0
5	239.04	0
6	224.1	0
7	209.16	0

Stress Range Levels: 20 Omitted cycles: 0

a) The stress range exceedance diagram (stress spectrum)





b) the stress range frequency distribution diagram

Day 1

The End