Multiaxial Fatigue

Professor Darrell Socie

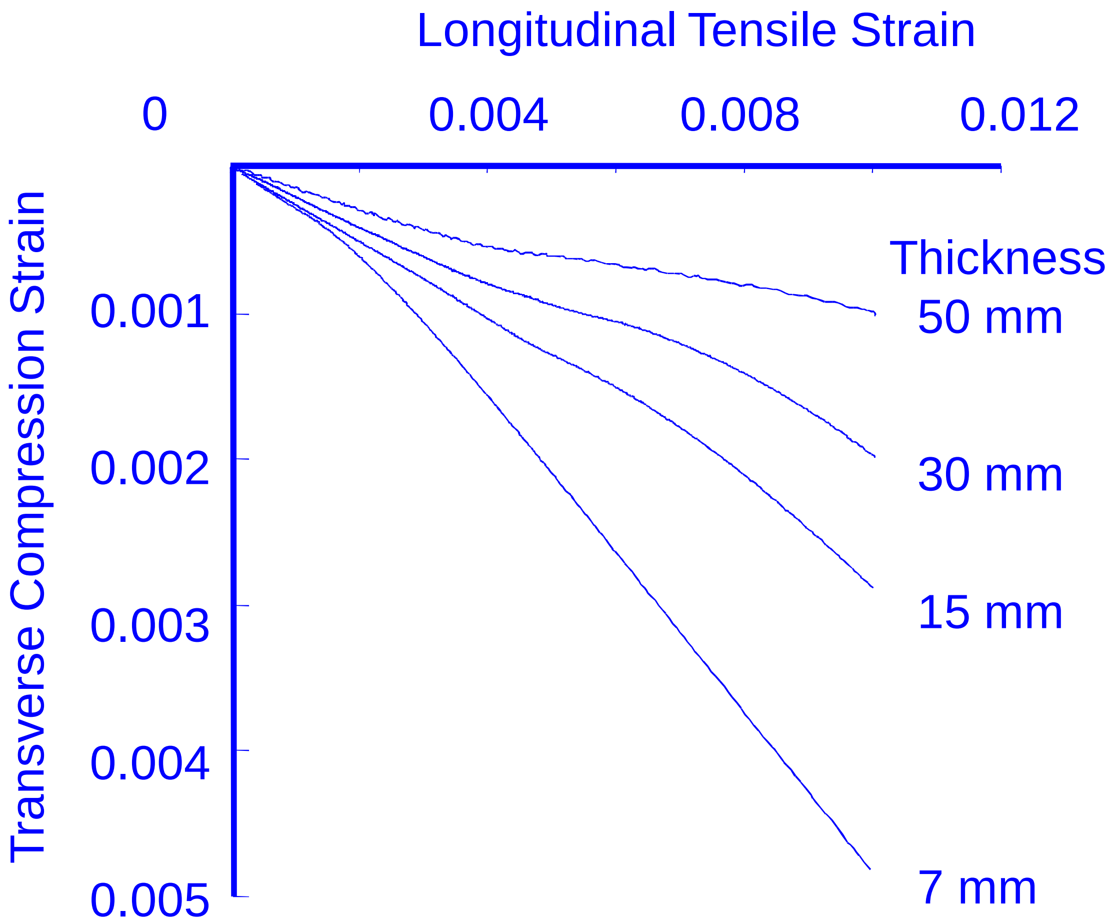
Outline

- Stresses around holes
- Crack Nucleation
- Crack Growth

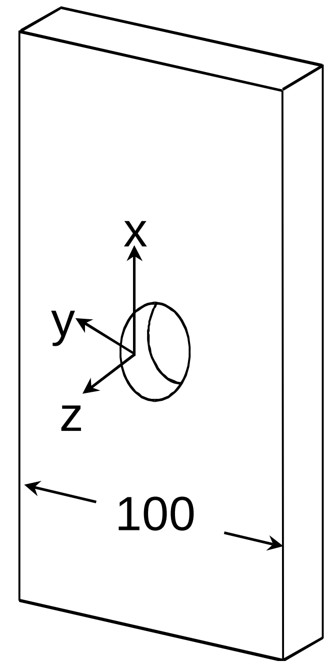
Multiaxial Fatigue Problems

- Uniaxial loading that produces multi stressesund stress concentrators
- Multiaxial loading that produces uniaxial stresses around stress concentrators
- Multiaxial loading that produces multiaxial stresses around stress concentrators
- Multiaxial loading that causes mixed mode long crack growth

3D stresses

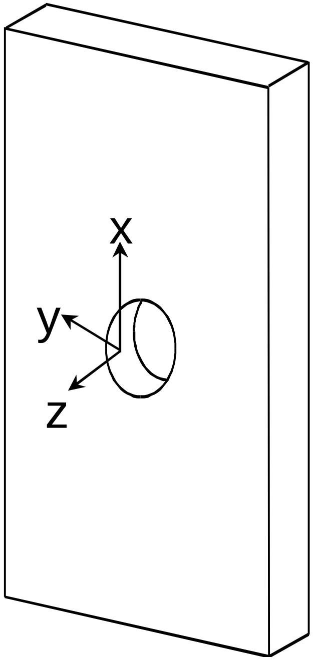


Longitudinal Tensile Strain
0
0.004
0.008
0.012
Transverse Compression Strain
0.001
0.002
0.003
0.004
0.005
Thickness
50 mm
30 mm
15 mm
7 mm



x
y
z
100

Notch Stresses



t	ε_x	ε_z	σ_x	σ_z
7	0.01	-0.005	63.5	0
15	0.01	-0.003	70.6	14.1
30	0.01	-0.002	73.0	21.8
50	0.01	-0.001	75.1	29.3

Multiaxial Fatigue Problems

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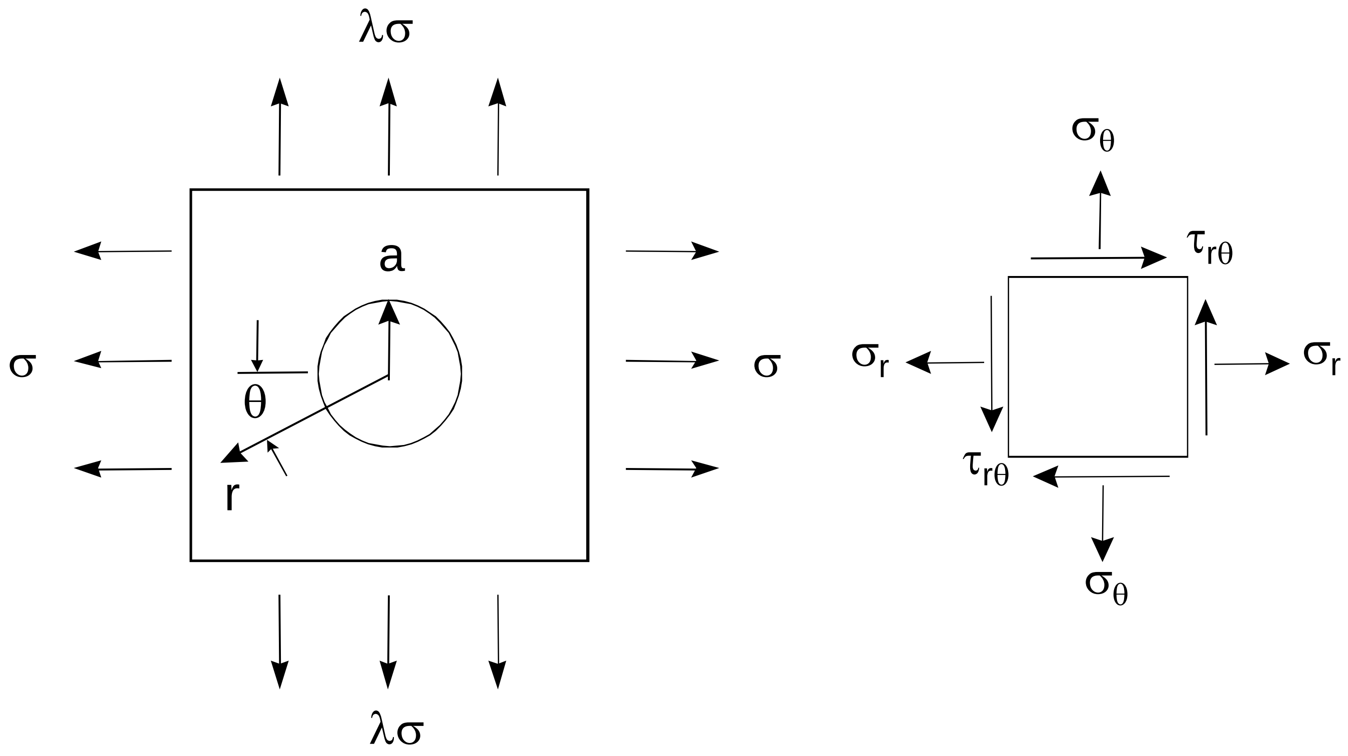
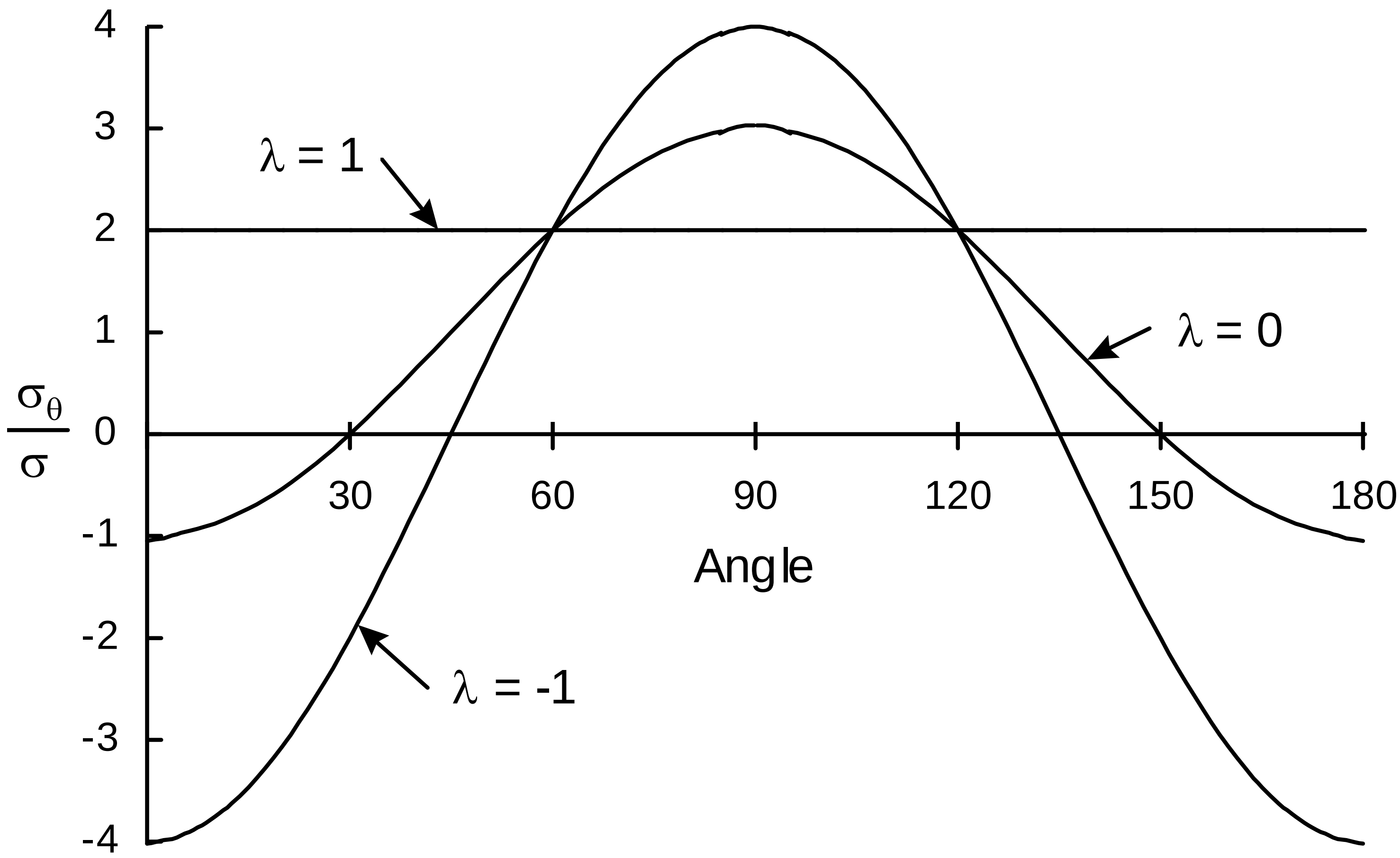
Hole in a Plate

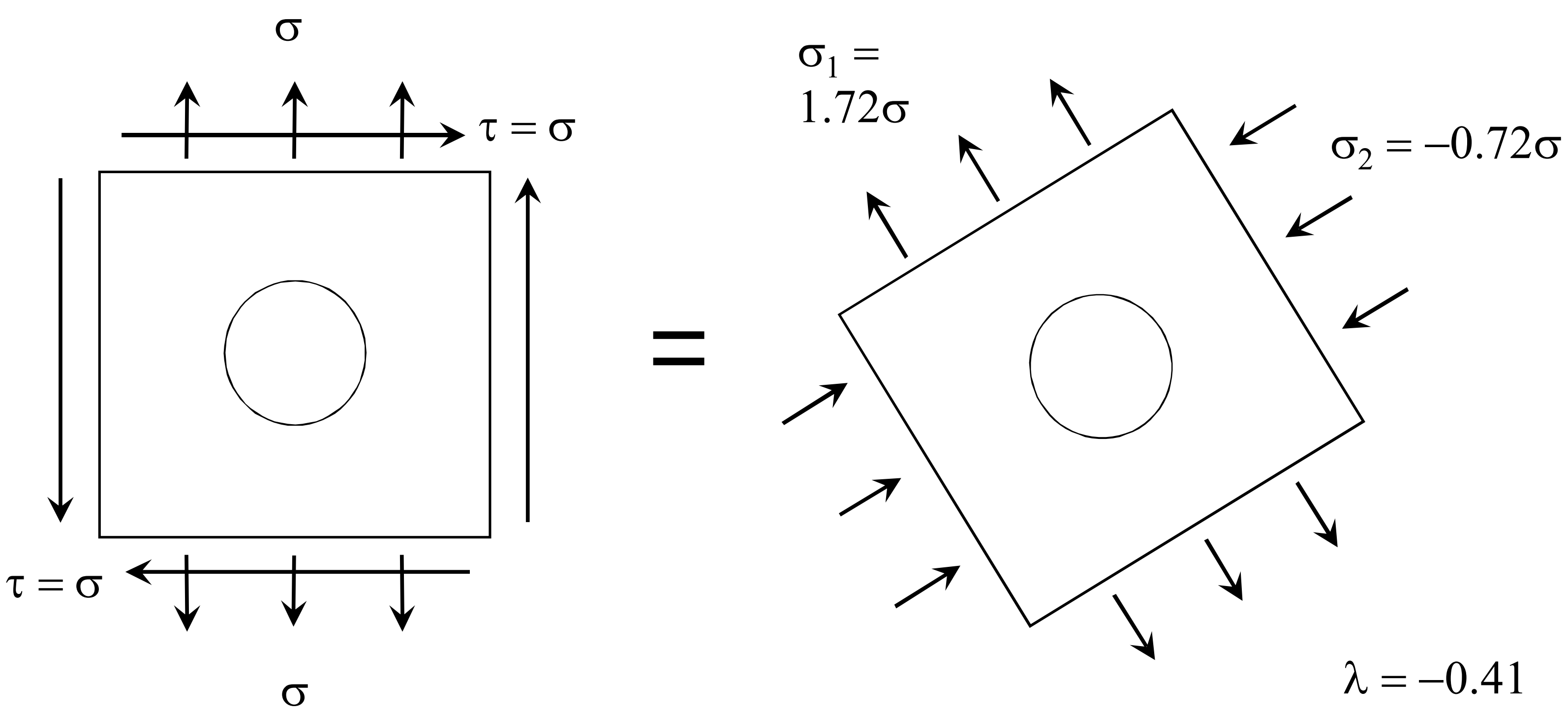


Stresses at the Hole

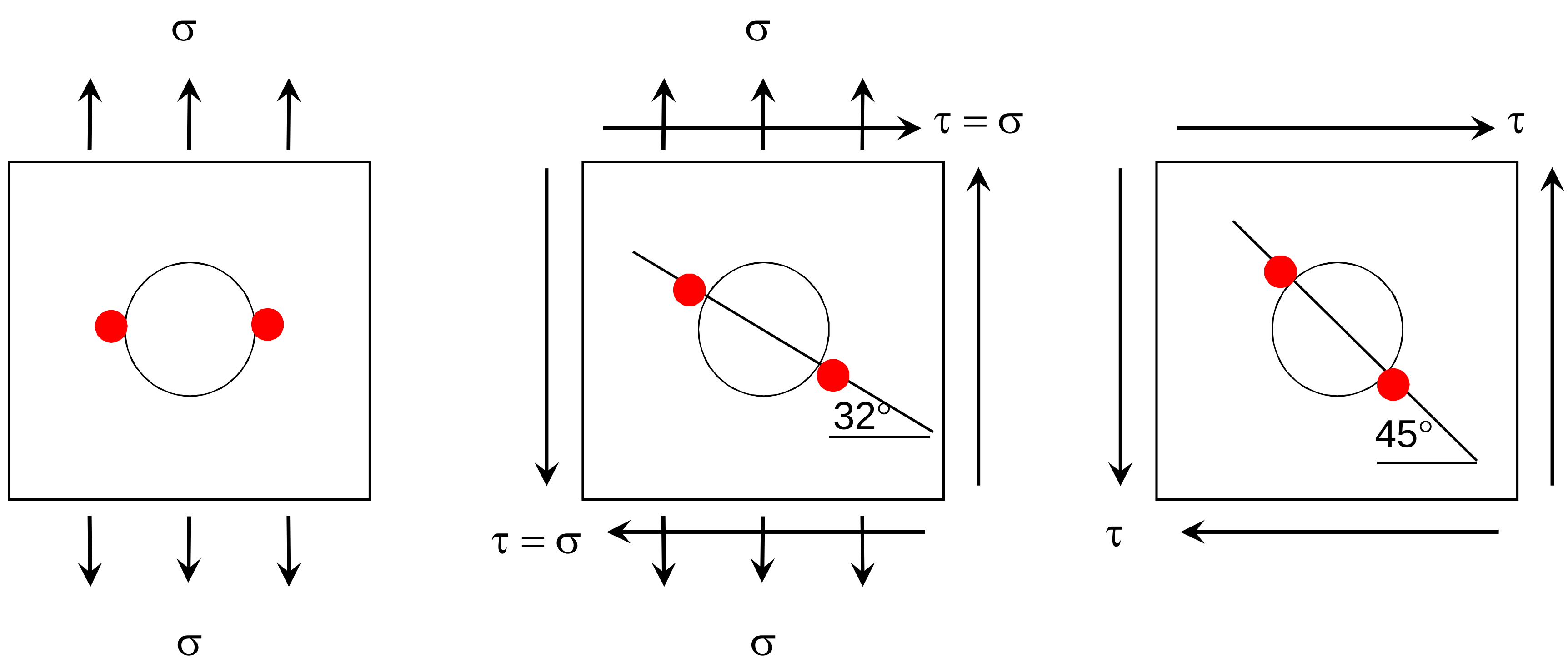


Stress concentration factor depends on type of loading

Combined Loading



Maximum Tensile Stress Location



$K_t = 3$

$\sigma_1 = \sigma$

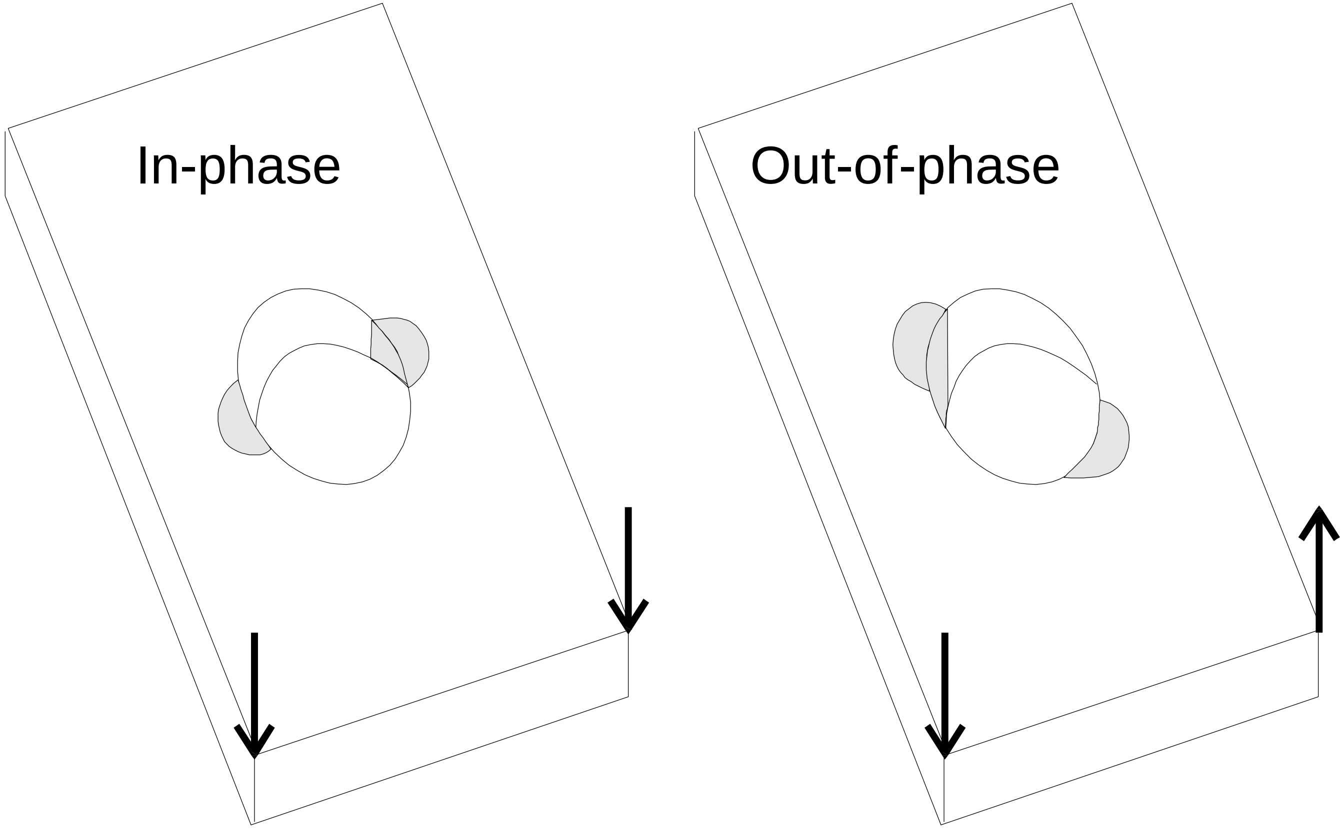
$K_t = 3.41$

$\sigma_1 = 1.72\sigma$

$K_t = 4$

$\sigma_1 = \tau$

In and Out of Phase Loading



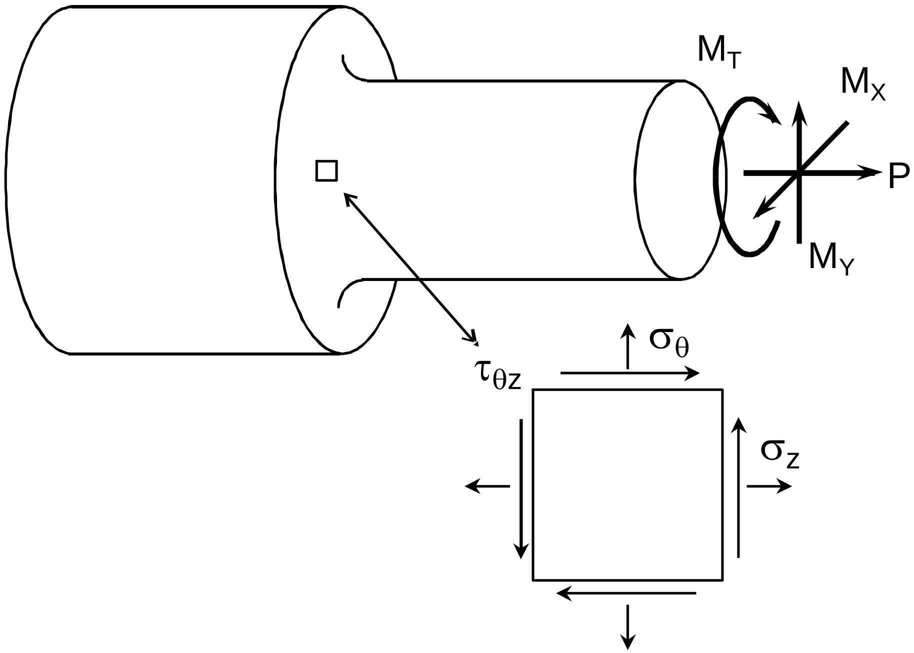
$K_t = 3$ $K_t = 4$

Damage location changes with load phasing

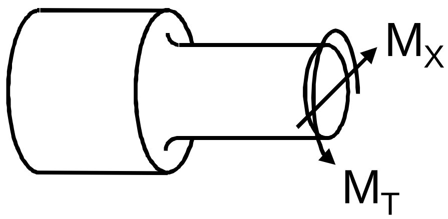
Multiaxial Fatigue Problems

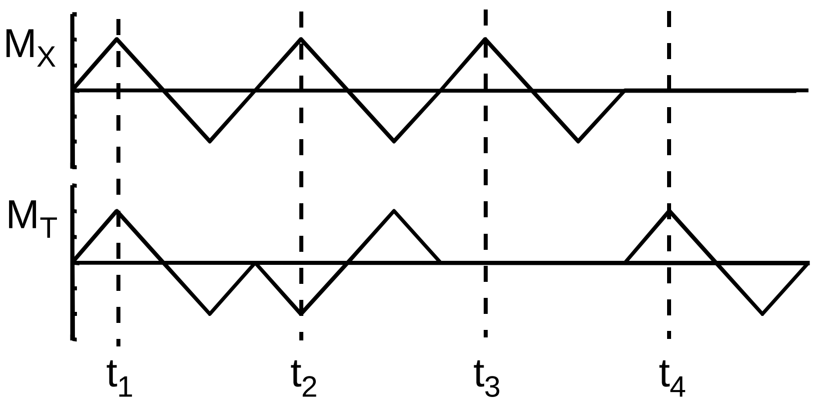
- Uniaxial loading that produces multiaxial stresses around stress concentrators
- Multiaxial loading that produces uniaxial stresses around stress concentrators
- Multiaxial loading that produces multiaxial stresses around stress concentrators
- Multiaxial loading that causes mixed mode long crack growth

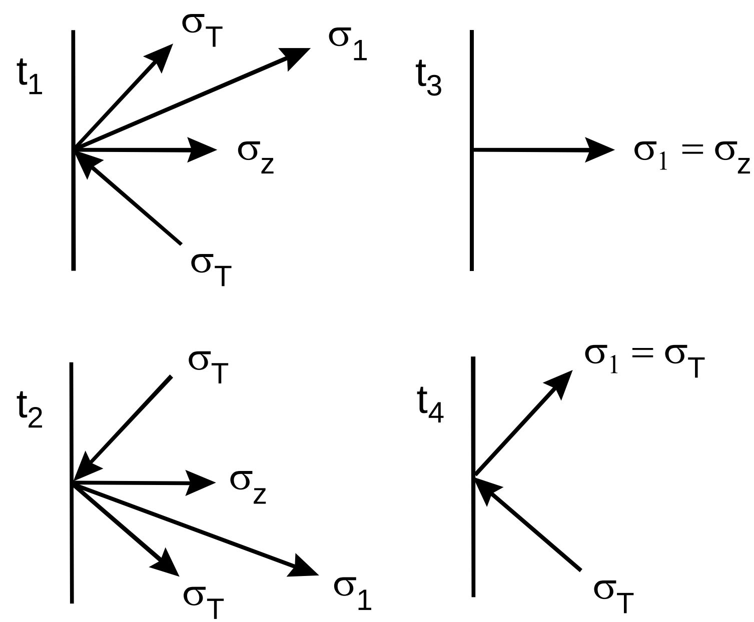
Notched Shaft Loading



Torsion Loading







Out-of-phase shear loading is needed to produce nonproportional stressing

Multiaxial Fatigue Problems

- Uniaxial loading that produces multiaxial stresses around stress concentrators
- Multiaxial loading that produces uniaxial stresses around stress concentrators
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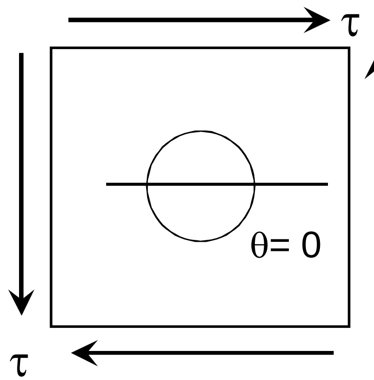
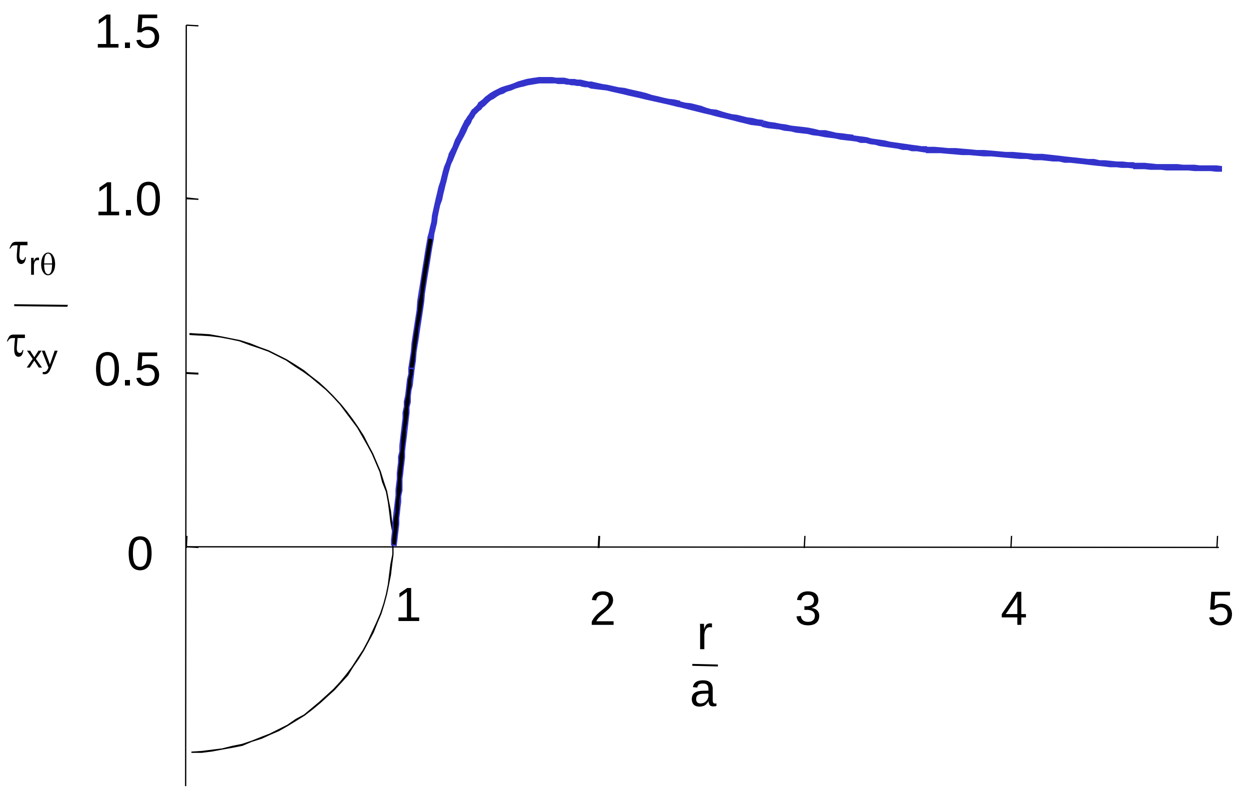
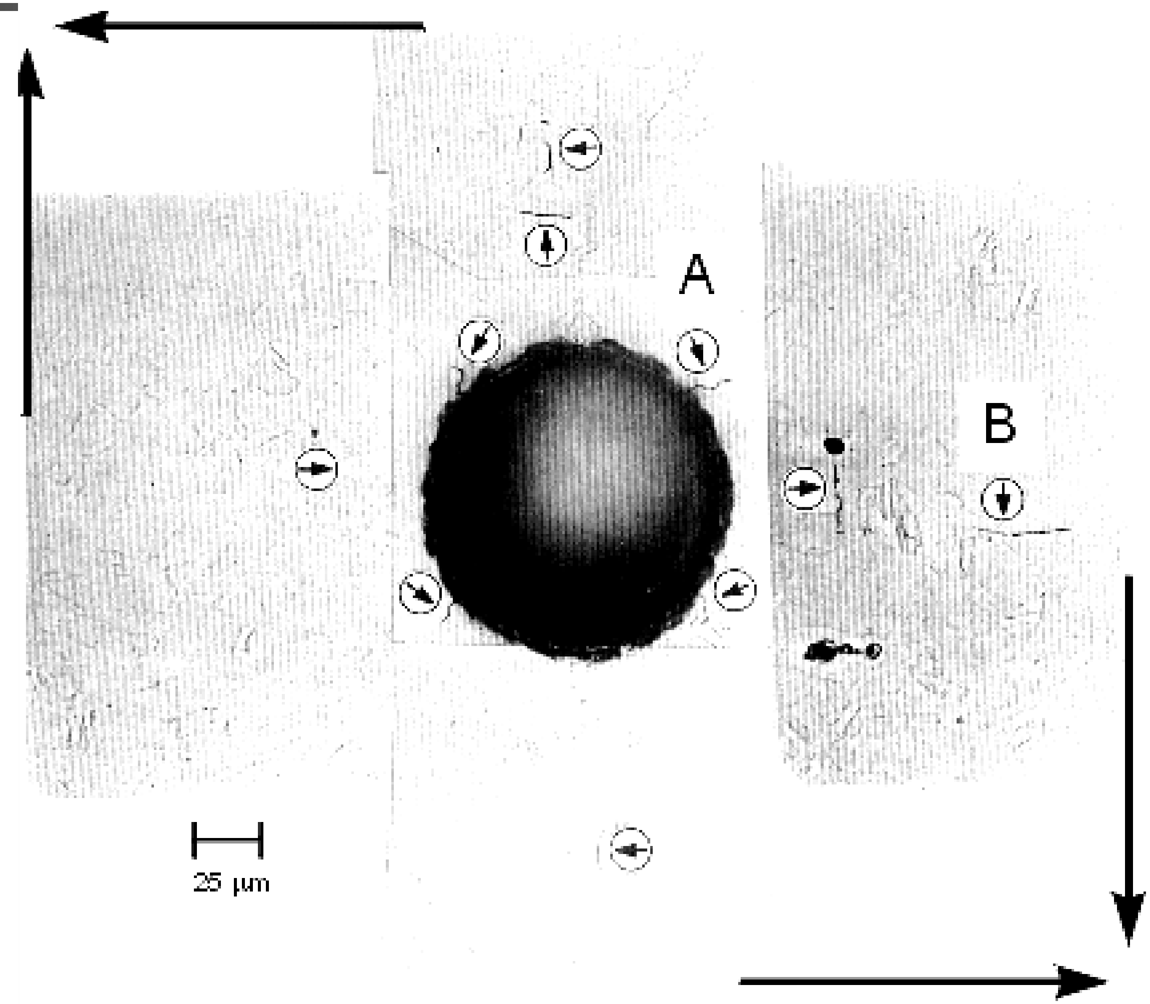
- Multiaxial loading that causes mixed mode long crack growth

Shear Stresses Around Hole θ = 0

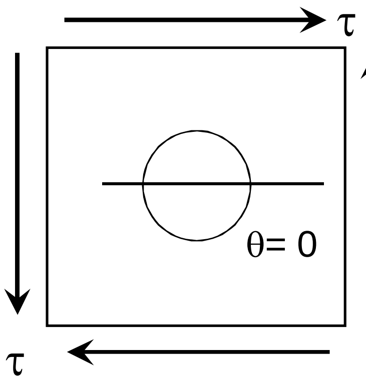


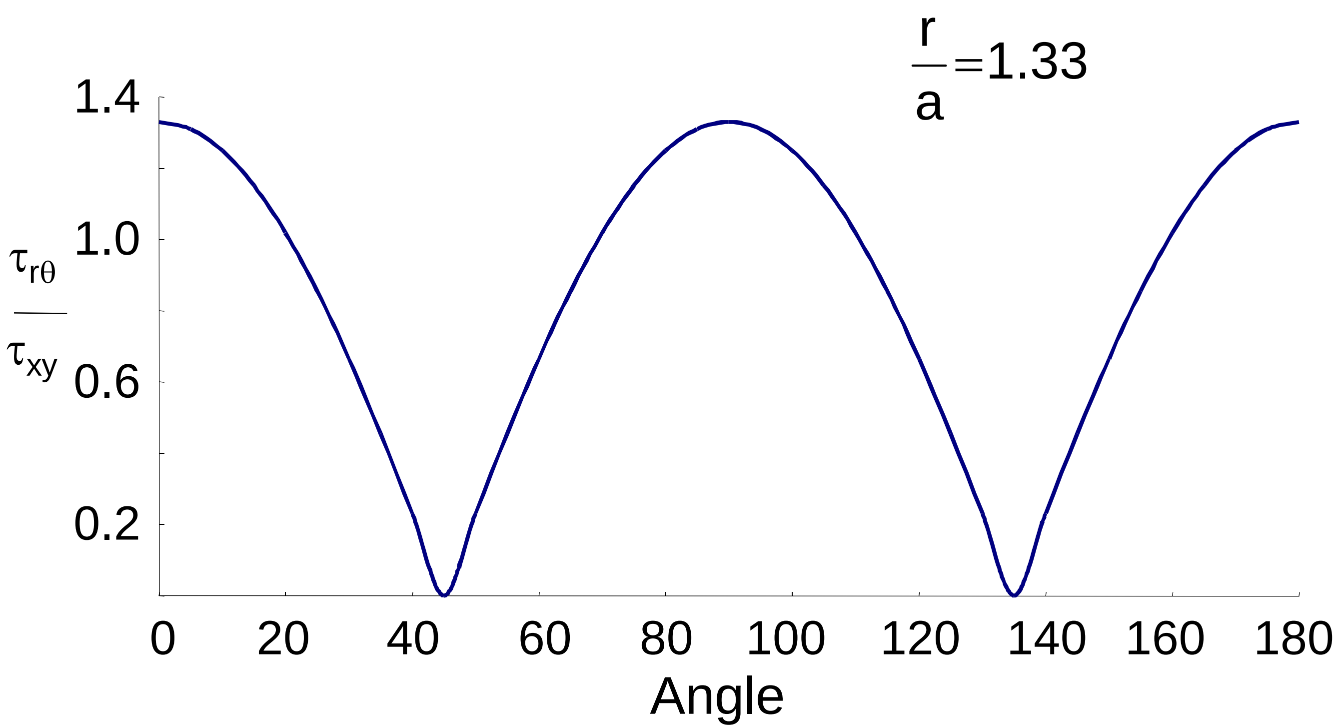


Torsion Experiments

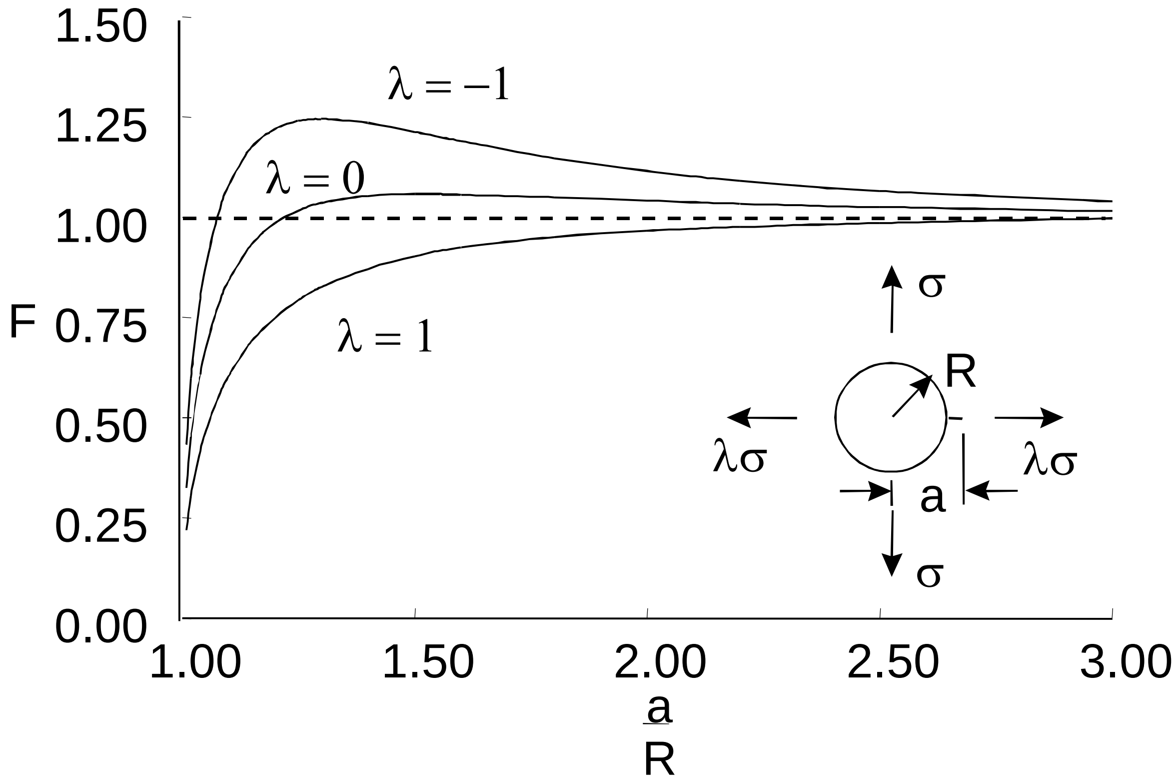


Shear Stresses Around Hole r = 1.33



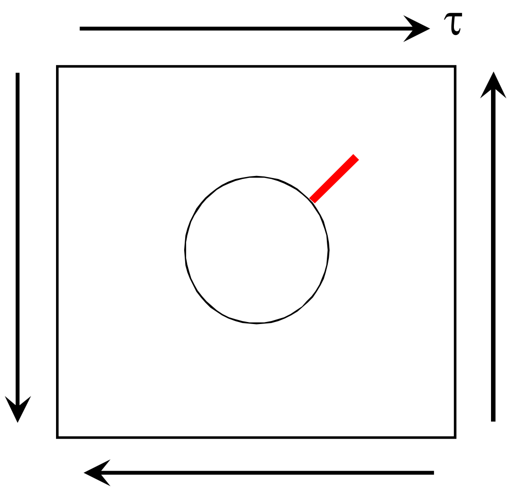


Stress Intensity Factors

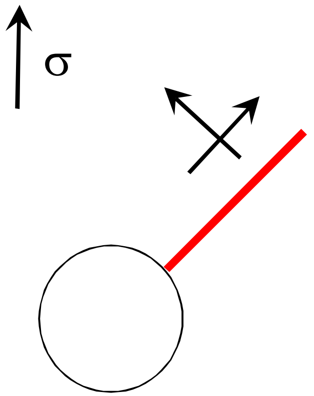


$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m \qquad K_I = F\,\Delta\sigma\sqrt{\pi a}$$

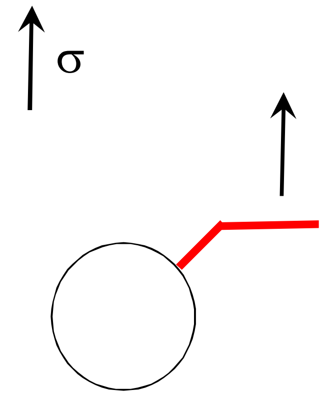
Cracks from Holes



Crack nucleates in shear



Mixed mode growth?



Tensile mode growth?

Summary

Cracks nucleate in a uniaxial stress field and then grow in a mixed tensile/shear stress field

Outline

- Stresses around holes
- Crack Nucleation
  - Stress Based Models
  - Strain Based Models
- Crack Growth

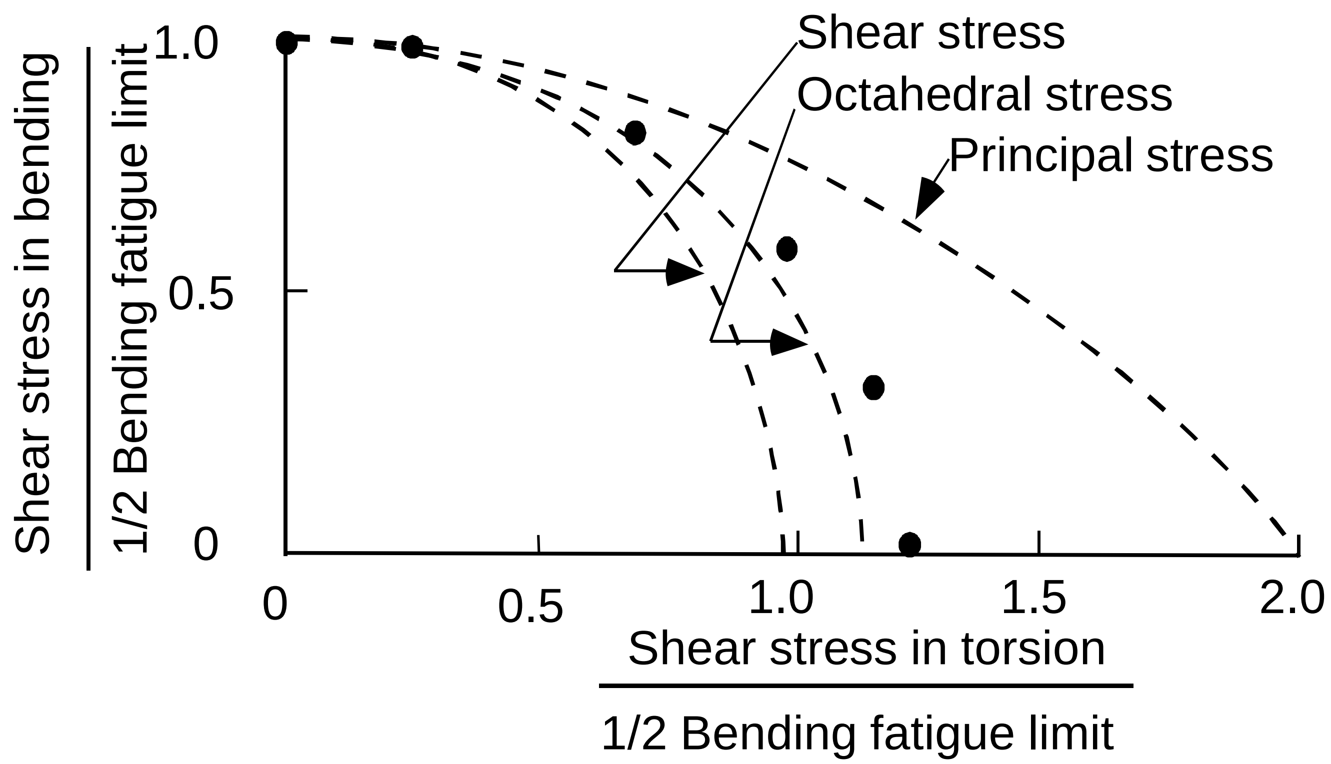
Fatigue Mechanisms Summary

- Fatigue cracks nucleate in shear
- Fatigue cracks grow in either shear or tension depending on material and state of stress

Stress Based Models

- Sines
- Findley
- Dang Van

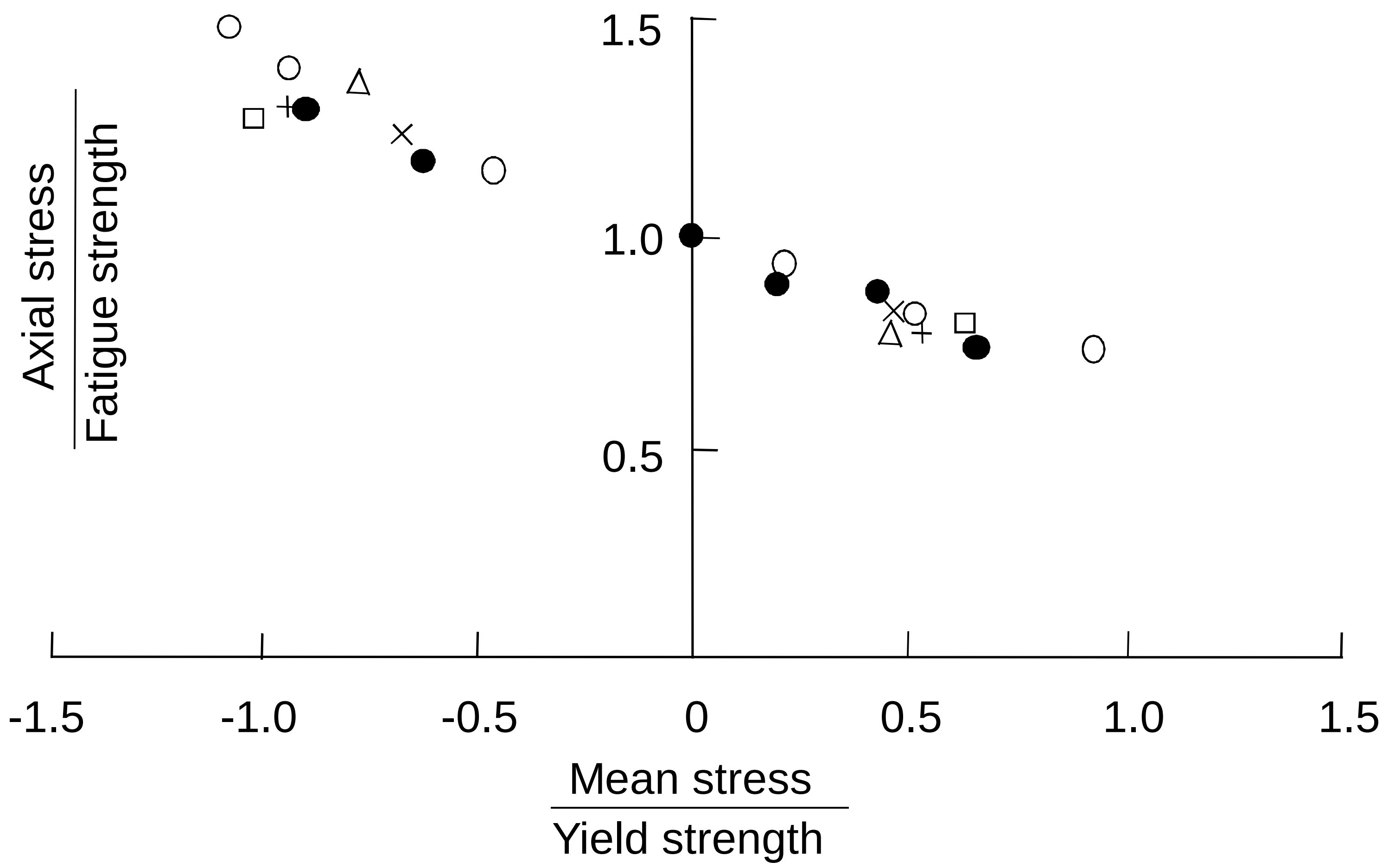
Bending Torsion Correlation



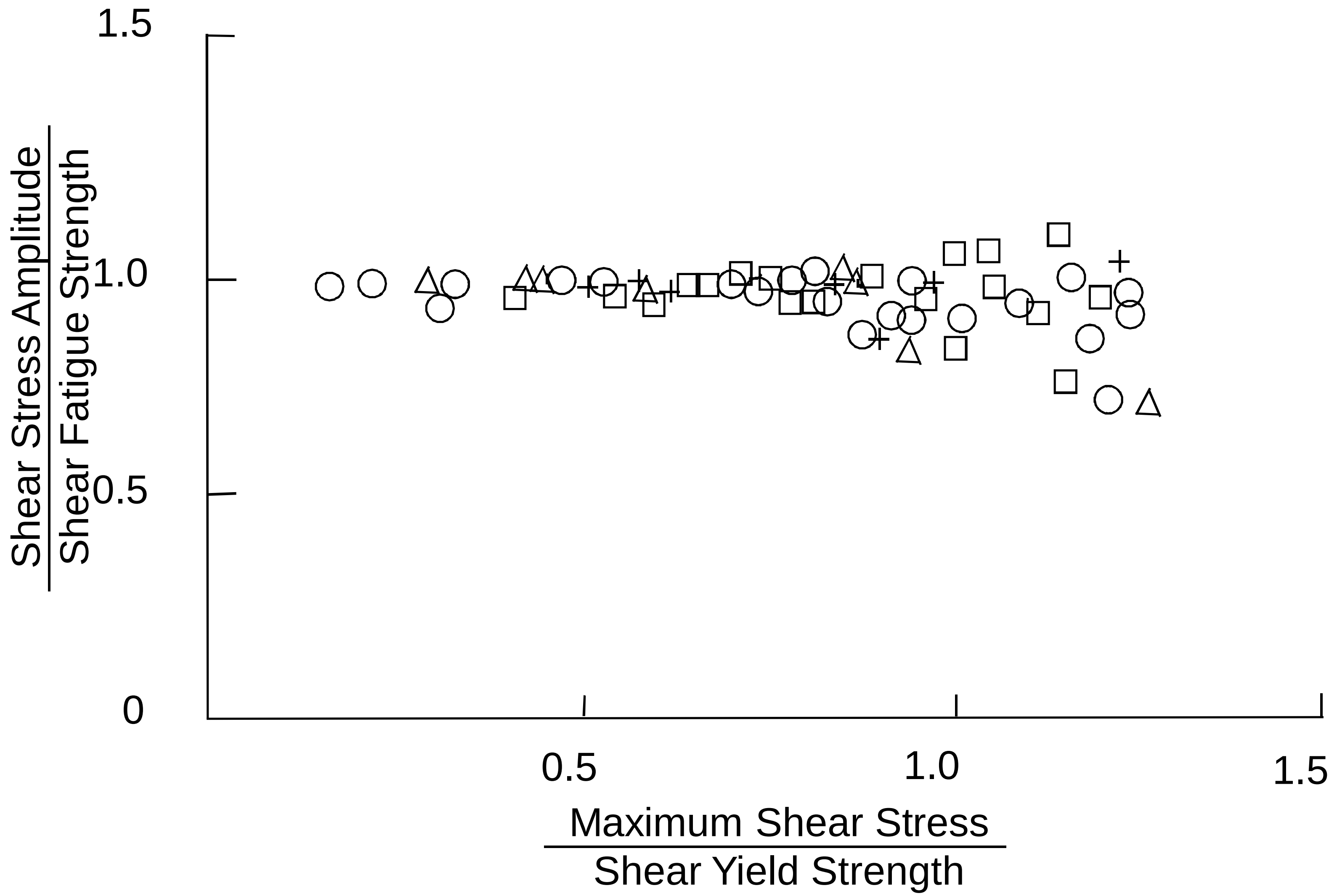
Test Results

- Cyclic tension with static tension
- Cyclic torsion with static torsion
- Cyclic tension with static torsion
- Cyclic torsion with static tension

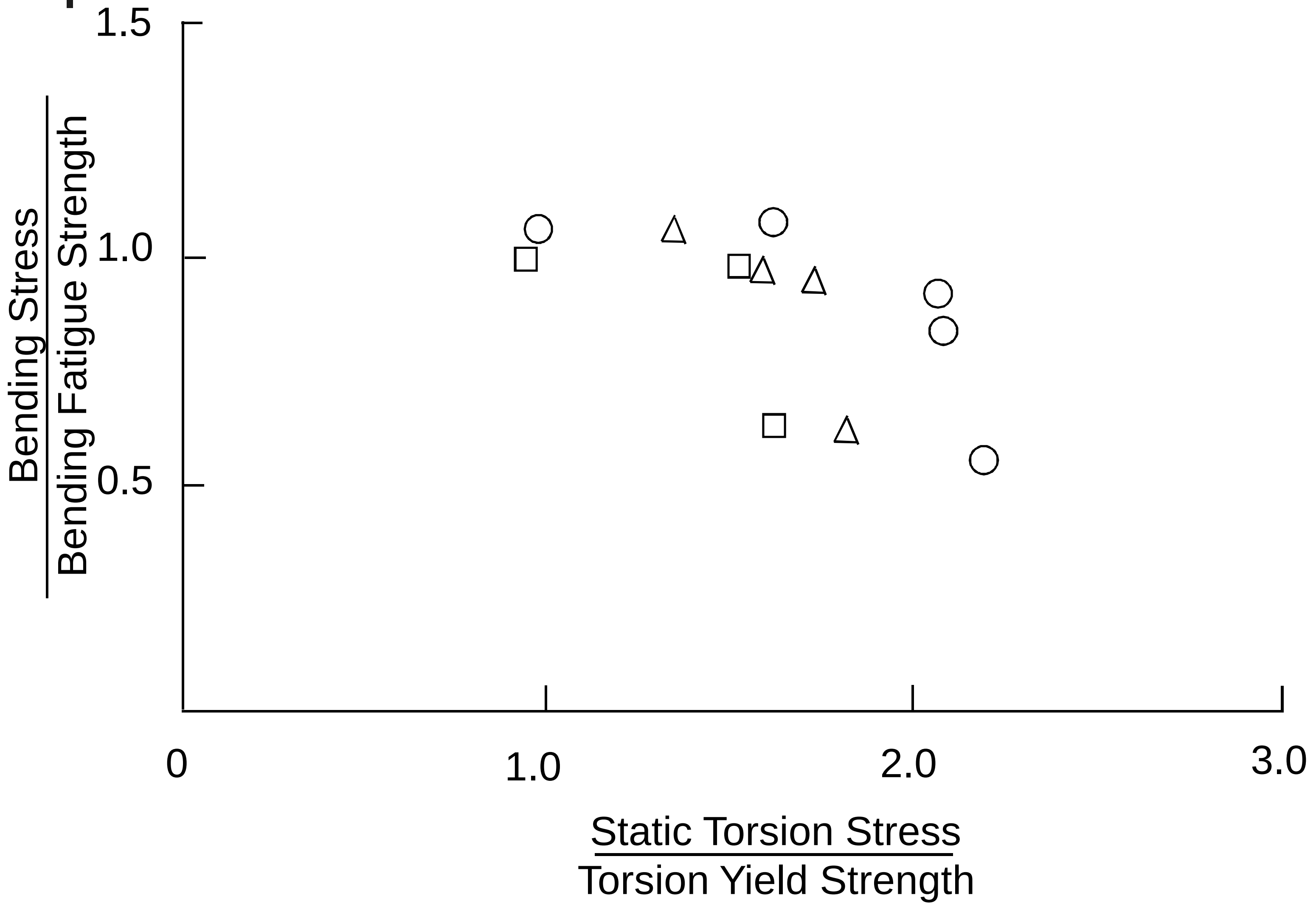
Cyclic Tension with Static Tension



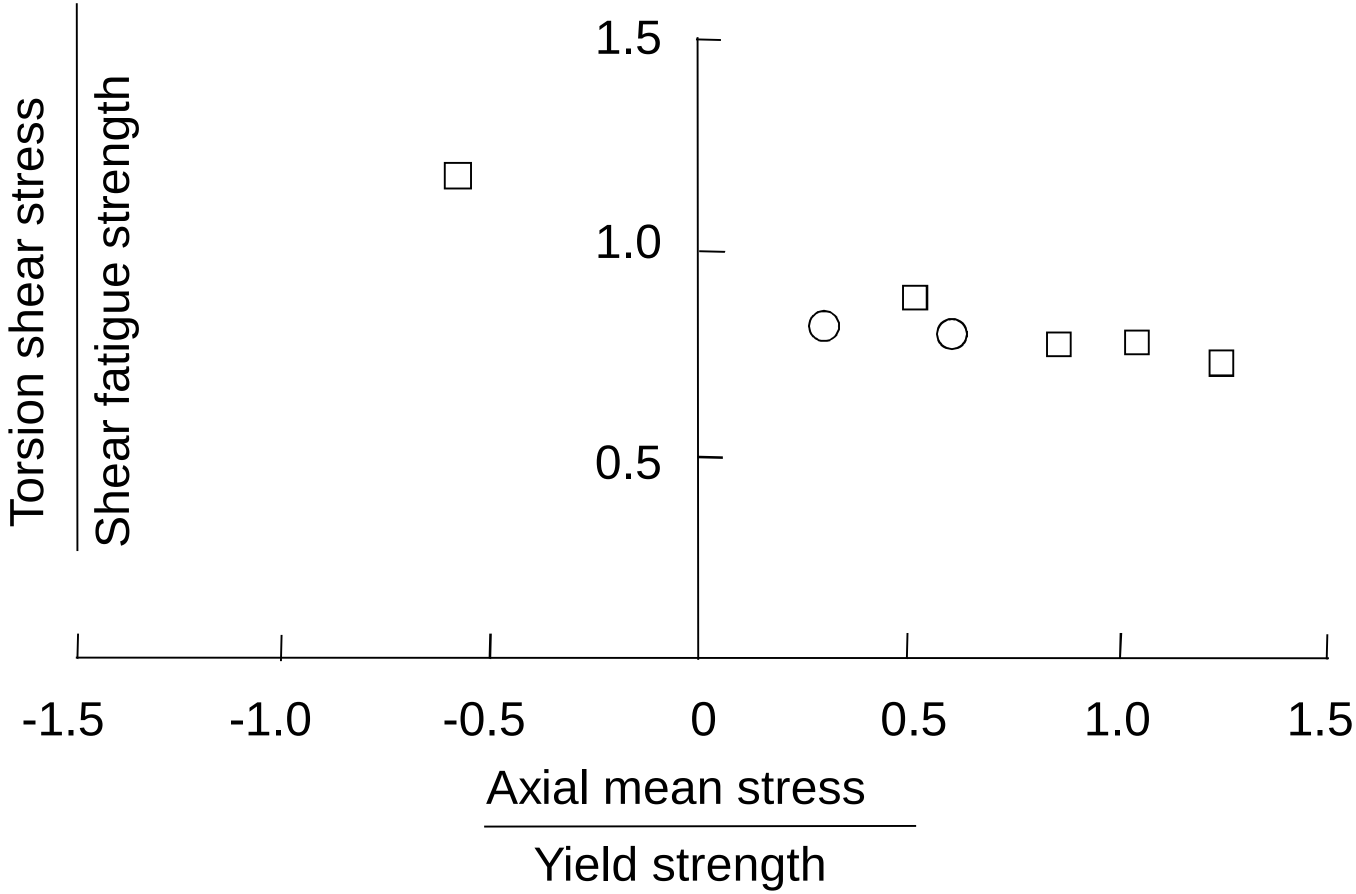
Cyclic Torsion with Static Torsion



Cyclic Tension with Static Torsion



Cyclic Torsion with Static Tension



Conclusions

- Tension mean stress affects both tension and torsion
- Torsion mean stress does not affect tension or torsion

Sines

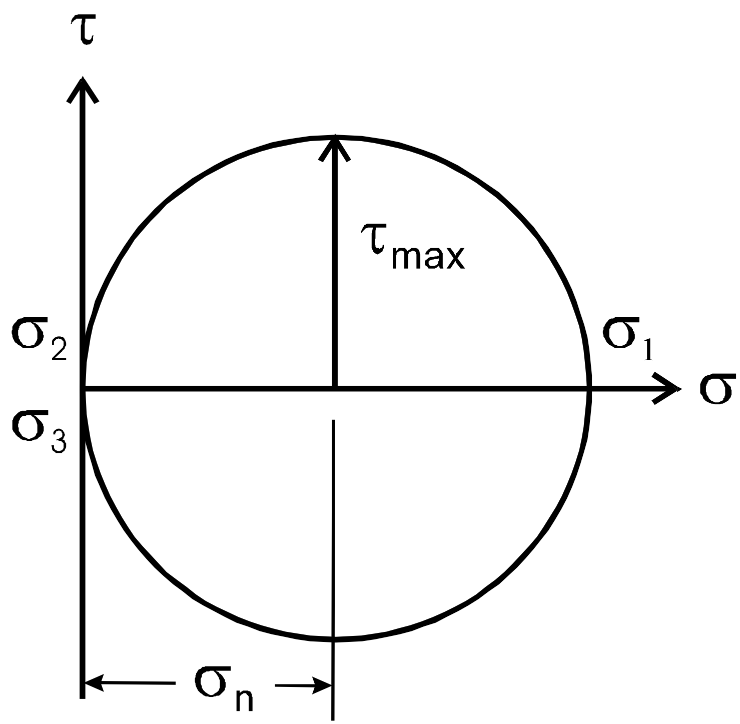
$$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$$

$$\frac{1}{6}\sqrt{(\Delta\sigma_x - \Delta\sigma_y)^2 + (\Delta\sigma_x - \Delta\sigma_z)^2 + (\Delta\sigma_y - \Delta\sigma_z)^2 + 6(\Delta\tau_{xy}^2 + \Delta\tau_{xz}^2 + \Delta\tau_{yz}^2)} +$$

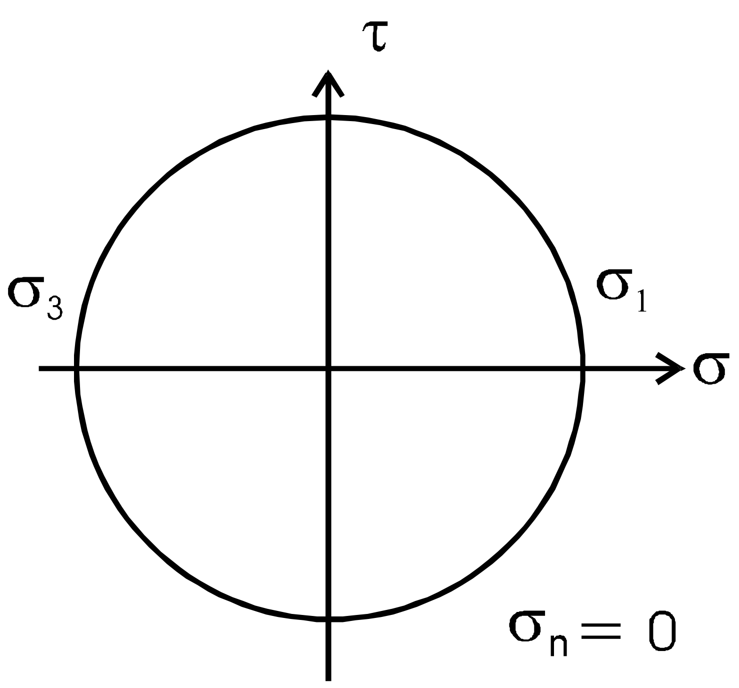
$$\alpha(\sigma_x^{mean} + \sigma_y^{mean} + \sigma_z^{mean}) = \beta$$

Findley

$$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

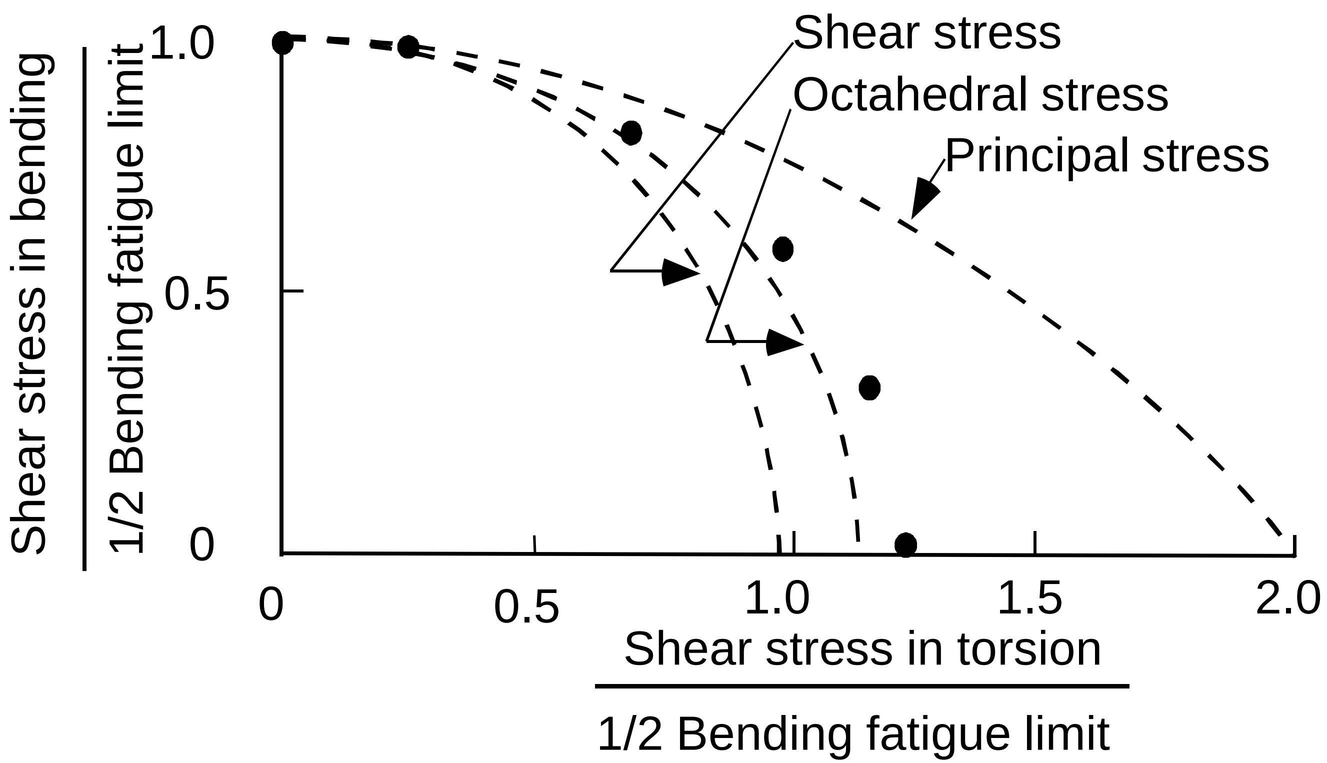


tension



torsion

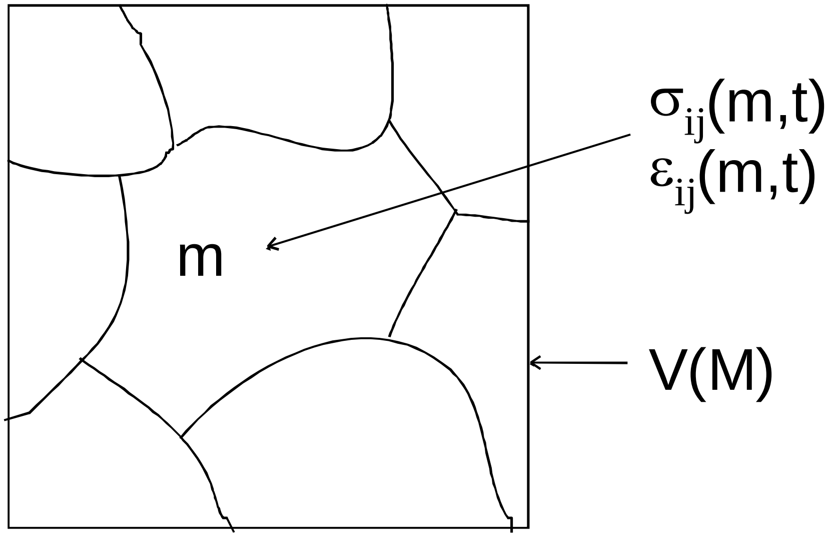
Bending Torsion Correlation



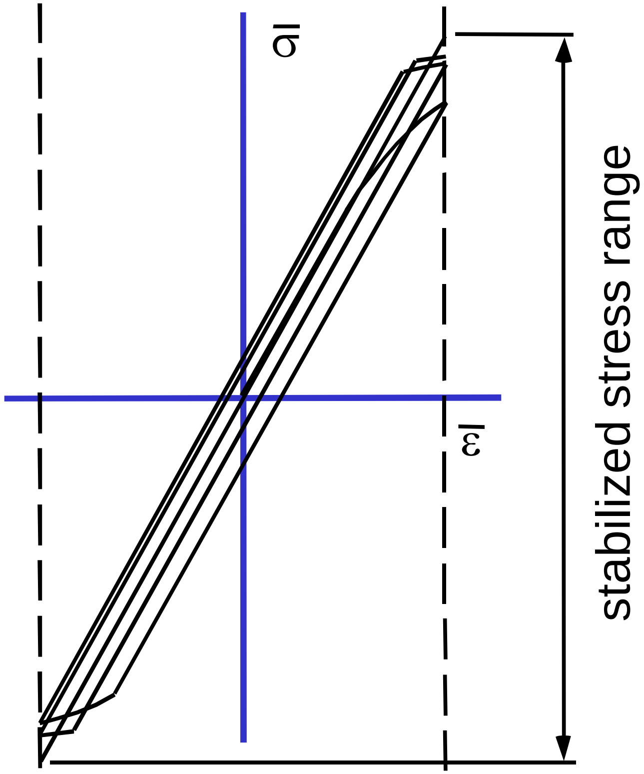
Dang Van

$$\tau(t) + a\sigma_h(t) = b$$

$\Sigma_{ij}(M,t)$ $E_{ij}(M,t)$

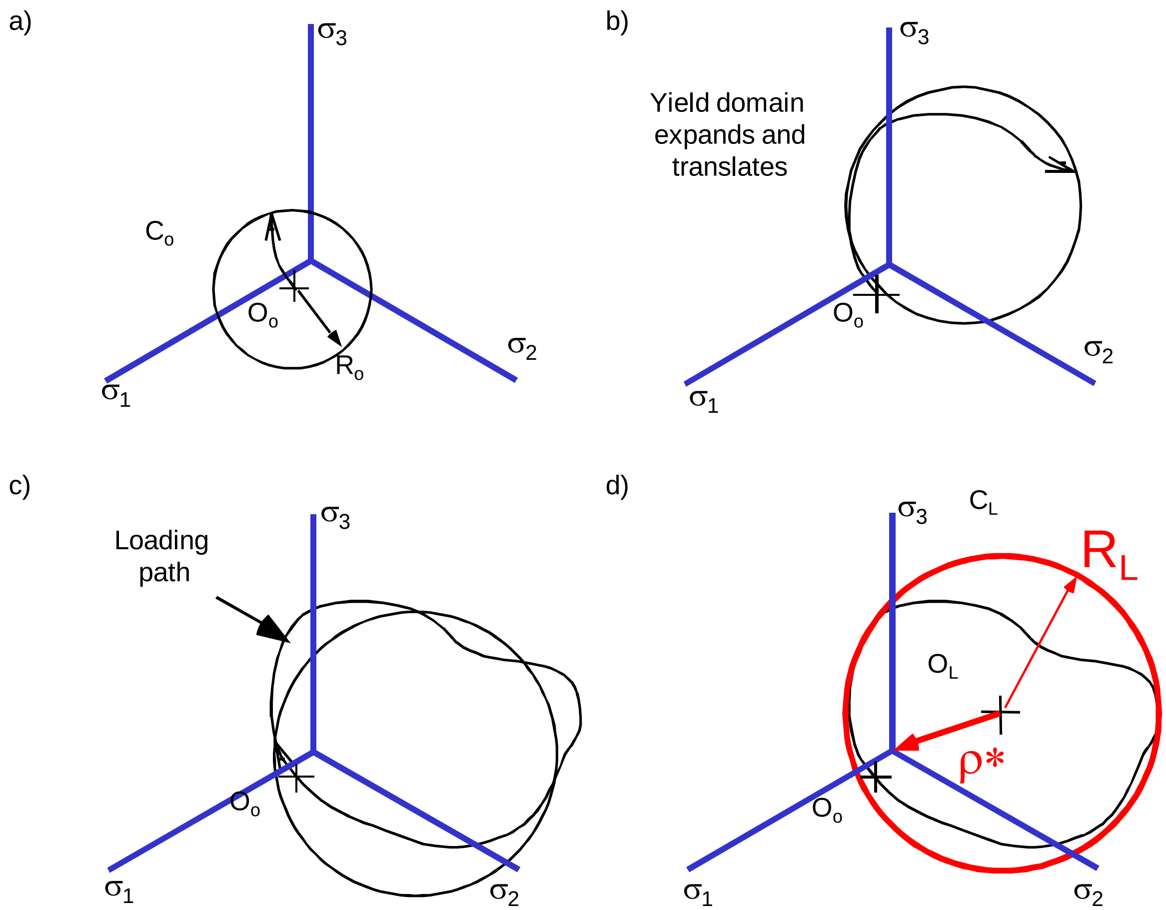


Isotropic Hardening



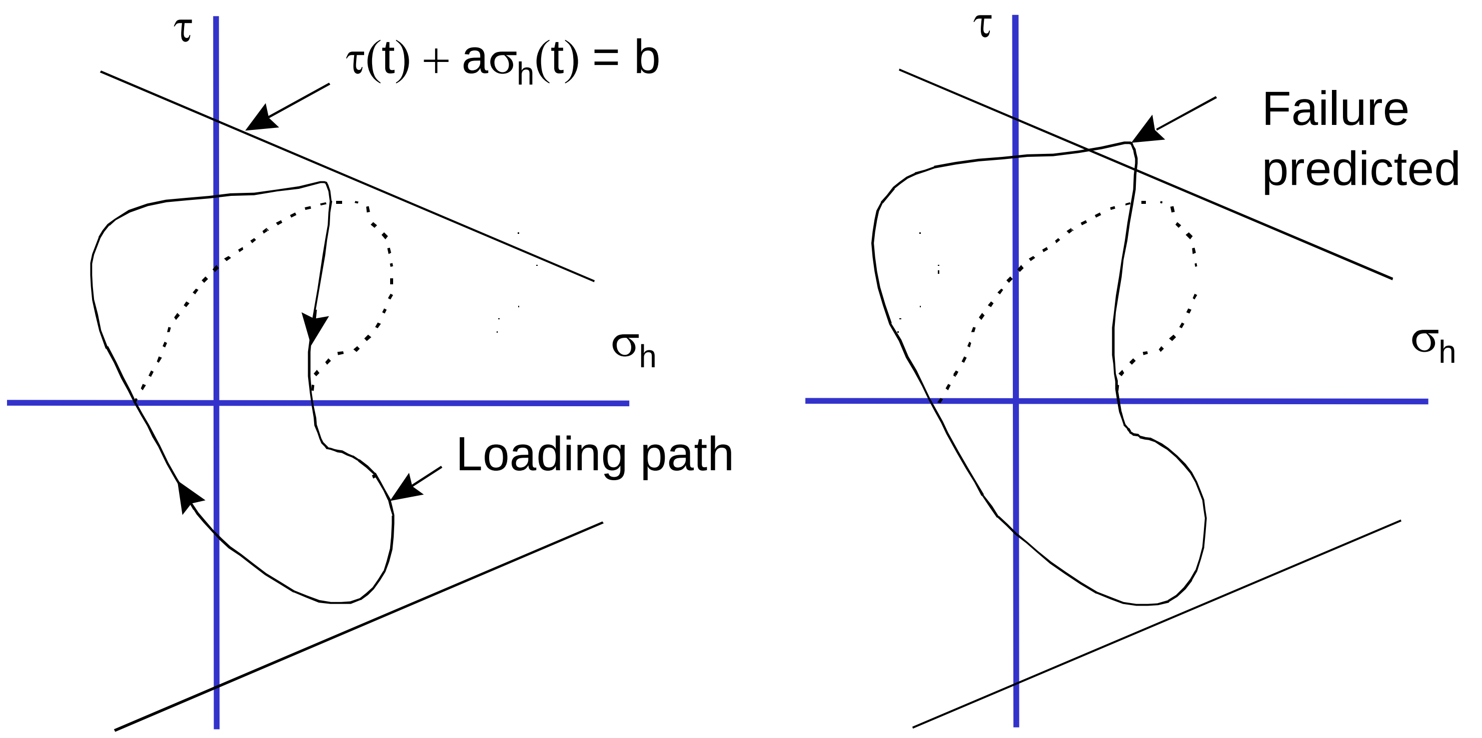
Failure occurs when the stress range is not elastic

Multiaxial Kinematic and Isotropic



ρ* stabilized residual stress

Dang Van (continued)



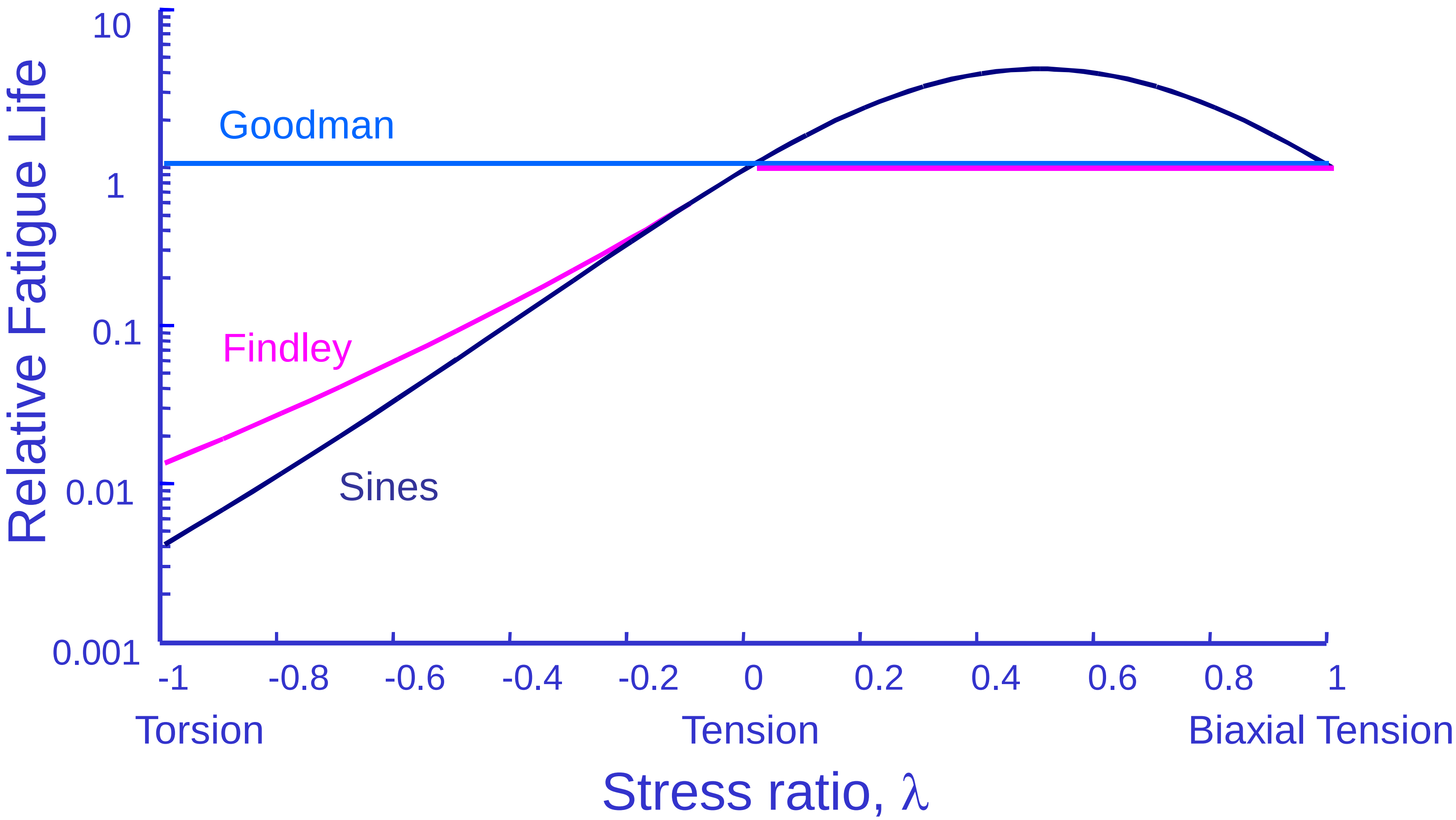
Stress Based Models Summary

Sines: $$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$$

Findley: $$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

Dang Van: $$\tau(t) + a\sigma_h(t) = b$$

Model Comparison R = -1



Outline

- Stresses around holes
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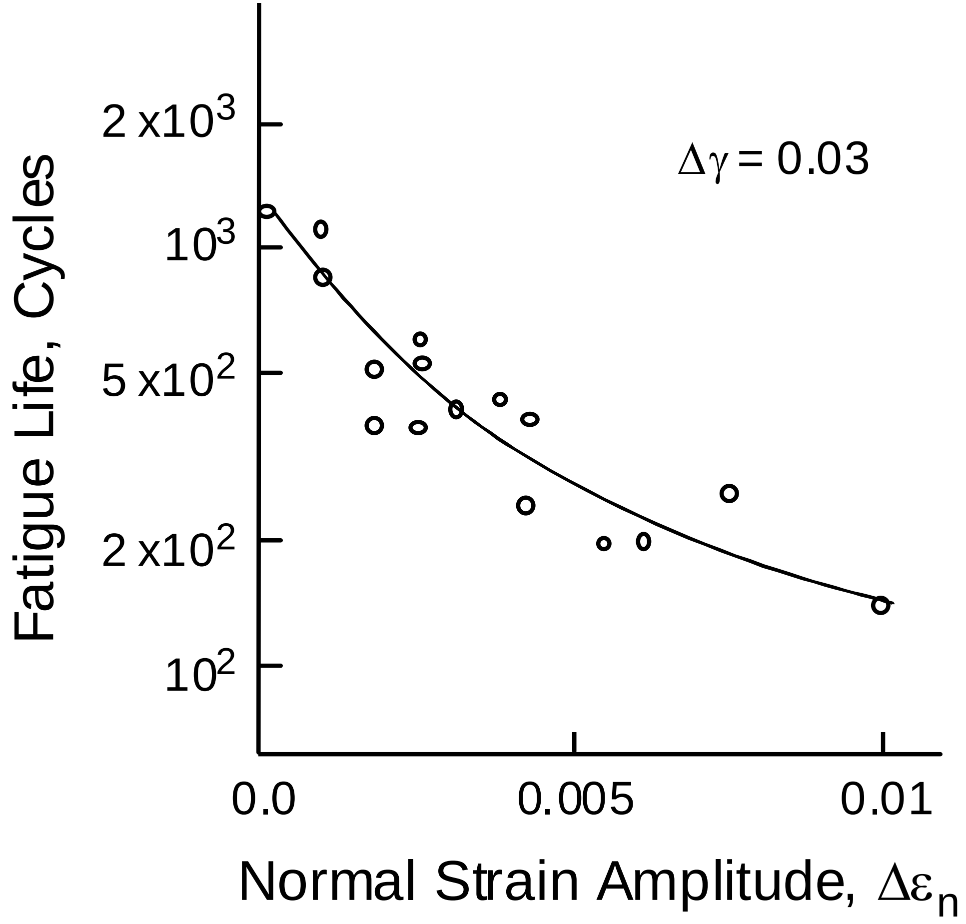
  - Stress Based Models
  - Strain Based Models
- Crack Growth

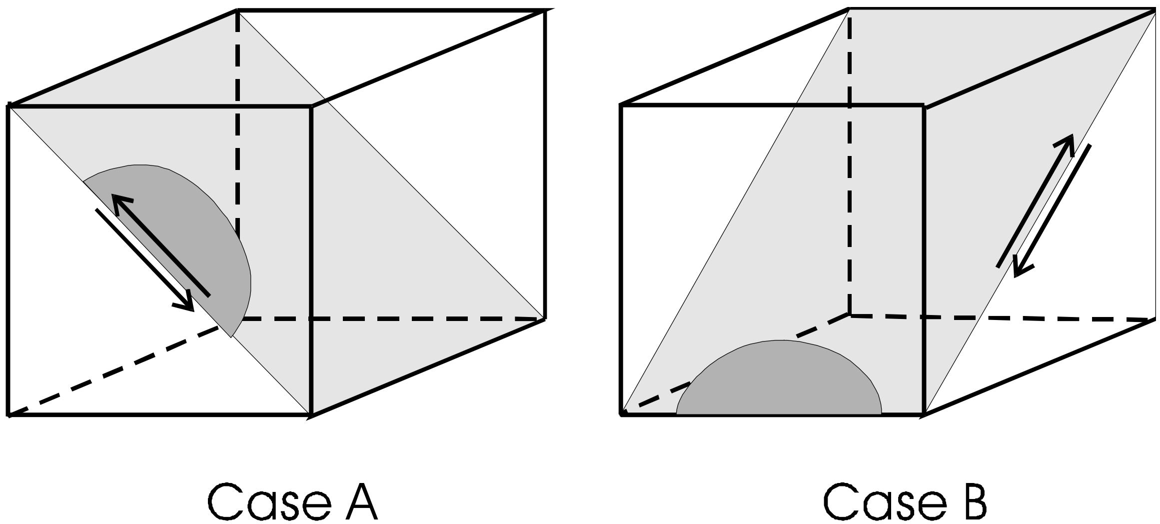
Strain Based Models

- Brown and Miller
- Fatemi and Socie
- Smith Watson and Topper

Brown and Miller



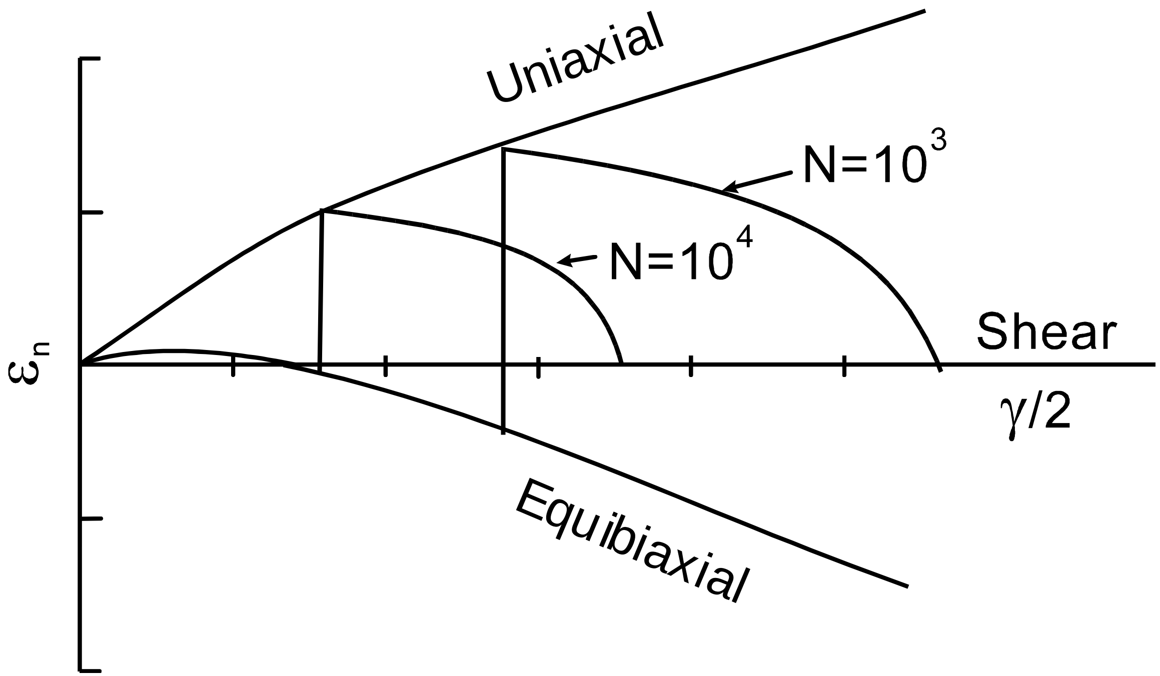
Case A and B



Growth along the surface

Growth into the surface

Brown and Miller (continued)

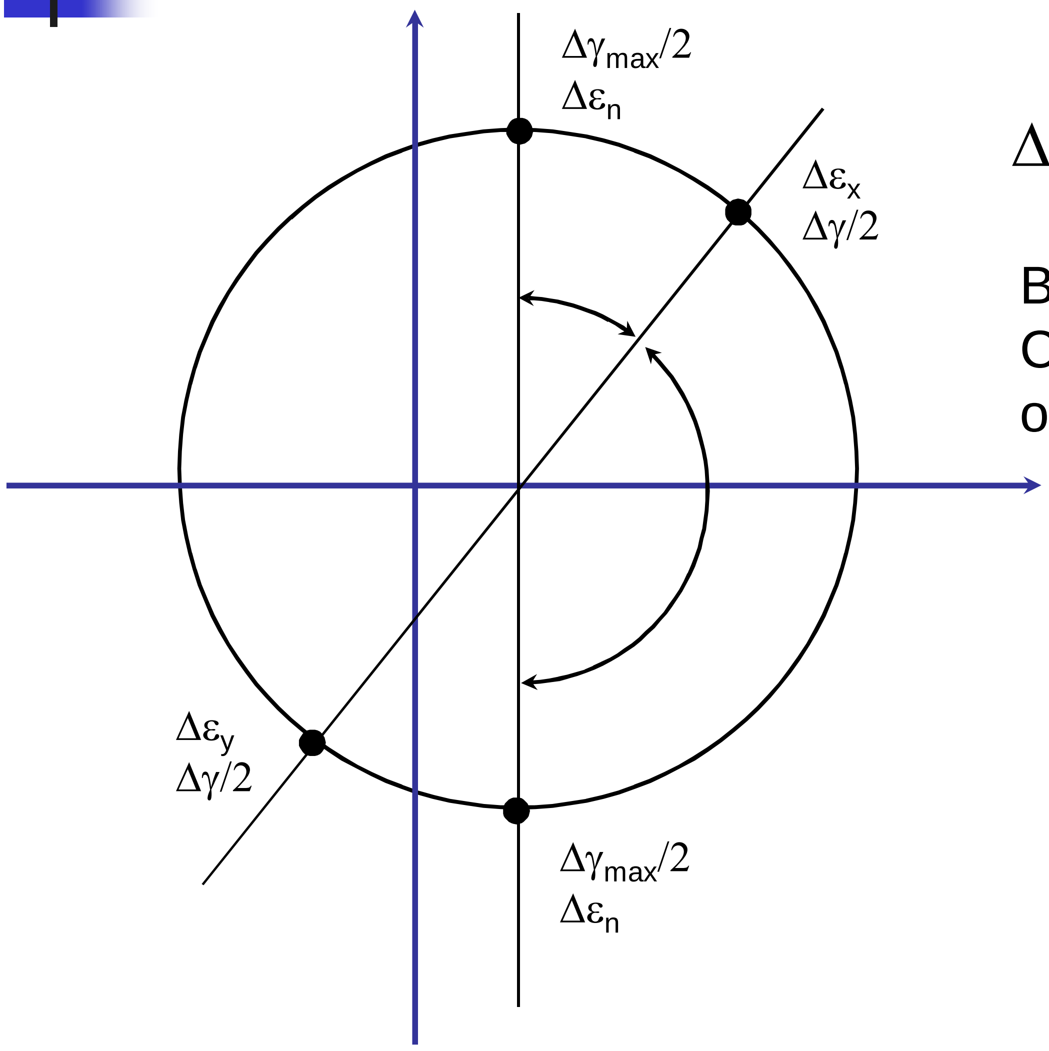


Brown and Miller (continued)

$$\Delta\hat{\gamma} = \left(\Delta\gamma_{max}^{\alpha} + S\Delta\varepsilon_{n}^{\alpha}\right)^{\frac{1}{\alpha}}$$

$$\frac{\Delta\gamma_{max}}{2} + S\,\Delta\varepsilon_{n} = A\frac{\sigma_{f}^{'} - 2\sigma_{n,mean}}{E}(2N_{f})^{b} + B\,\varepsilon_{f}^{'}(2N_{f})^{c}$$

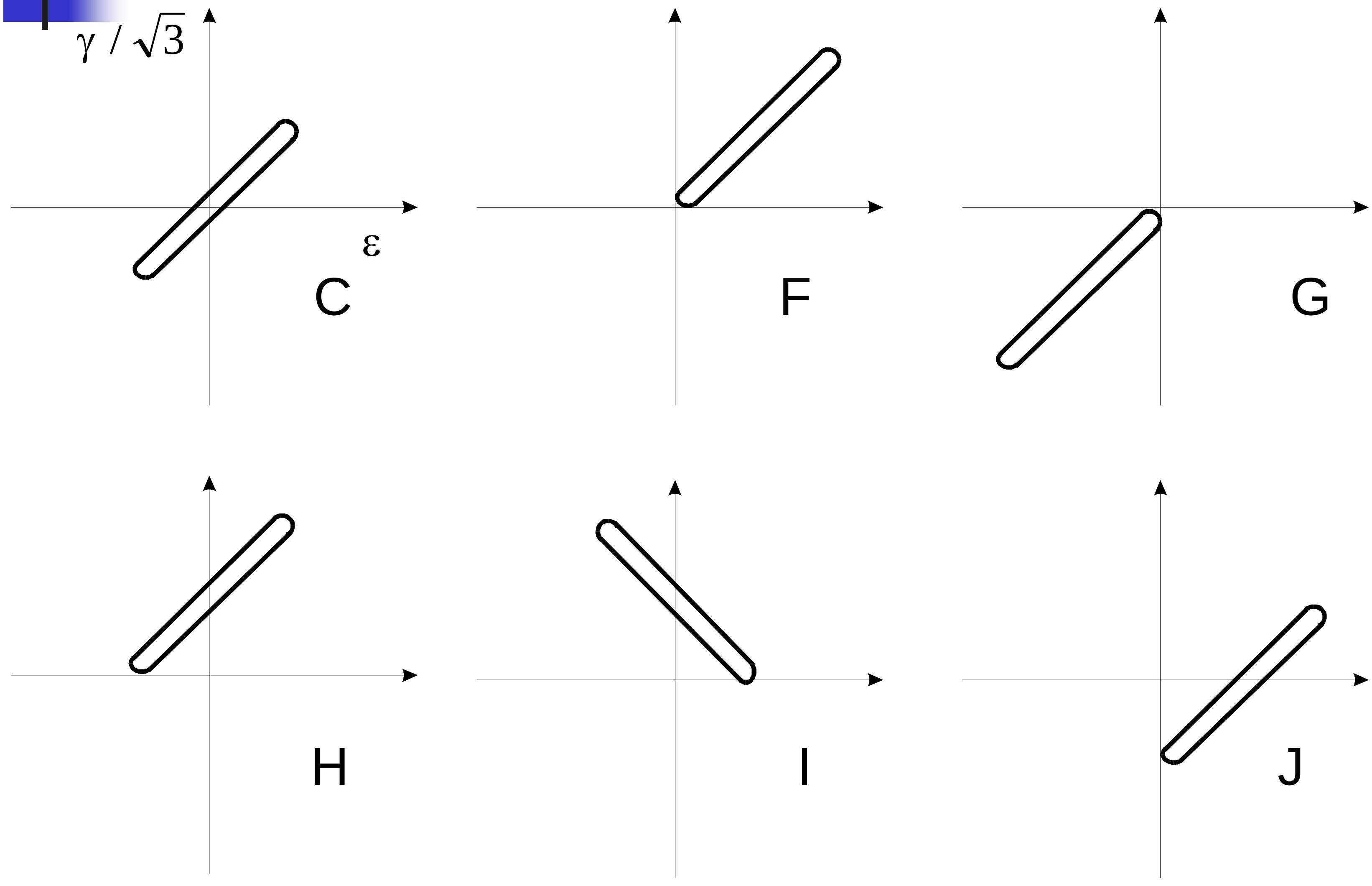
Mohr's Circle



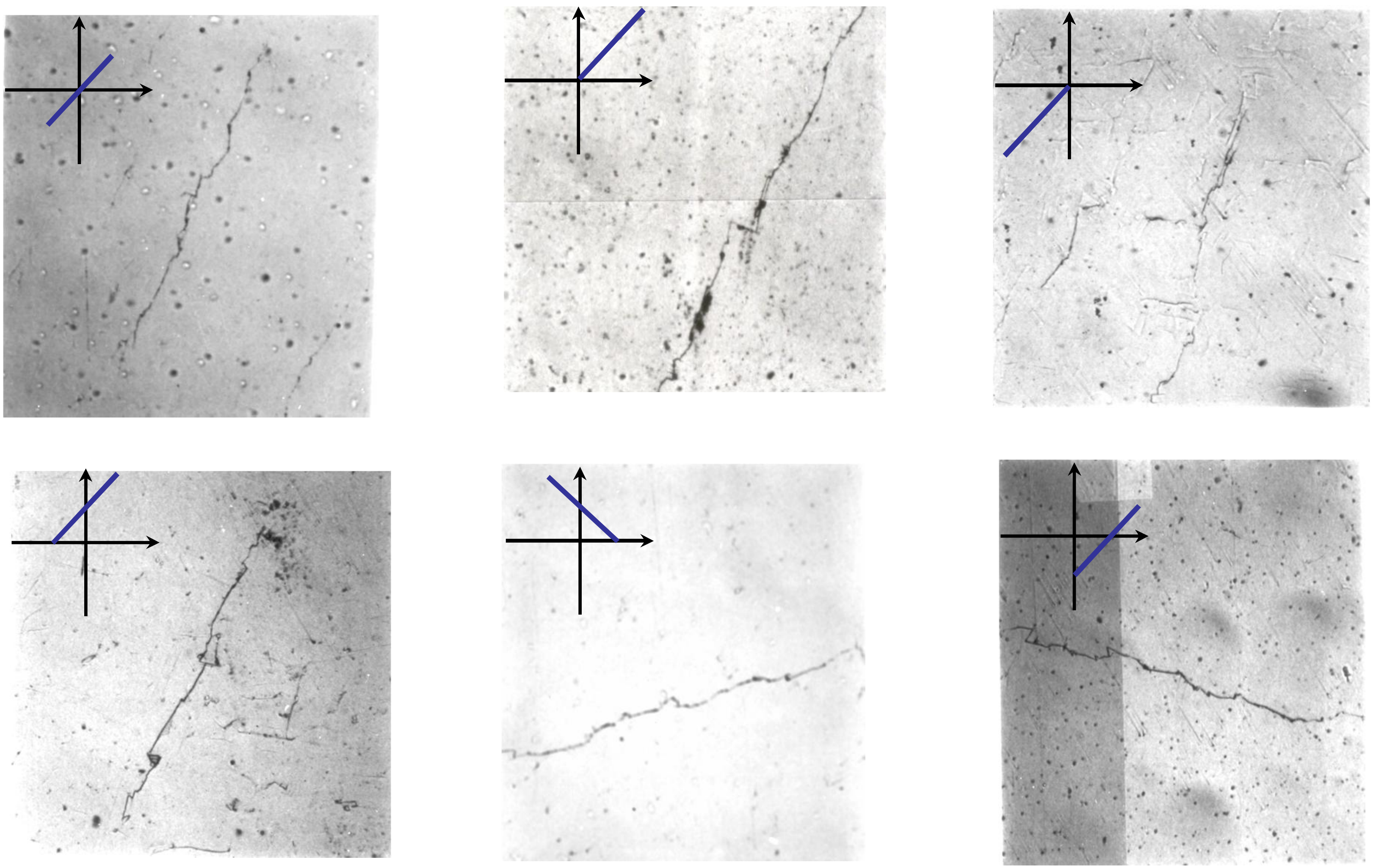
$$\Delta\hat{\gamma} = \left(\Delta\gamma_{max}^{\alpha} + S\Delta\varepsilon_{n}^{\alpha}\right)^{\frac{1}{\alpha}}$$

Brown and Miller -
Cracks should be equally likely on two planes 90° apart

Loading Histories



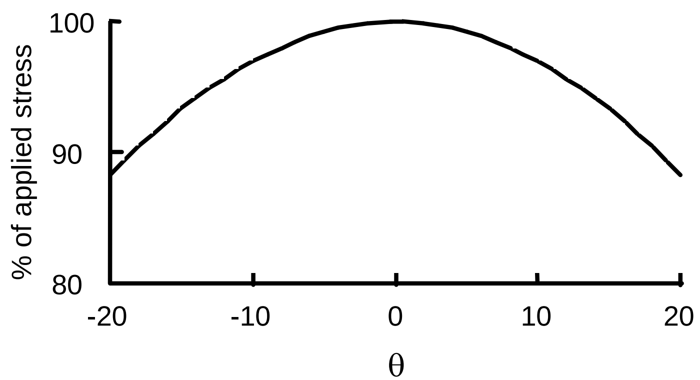
Crack Directions



Hypothesis

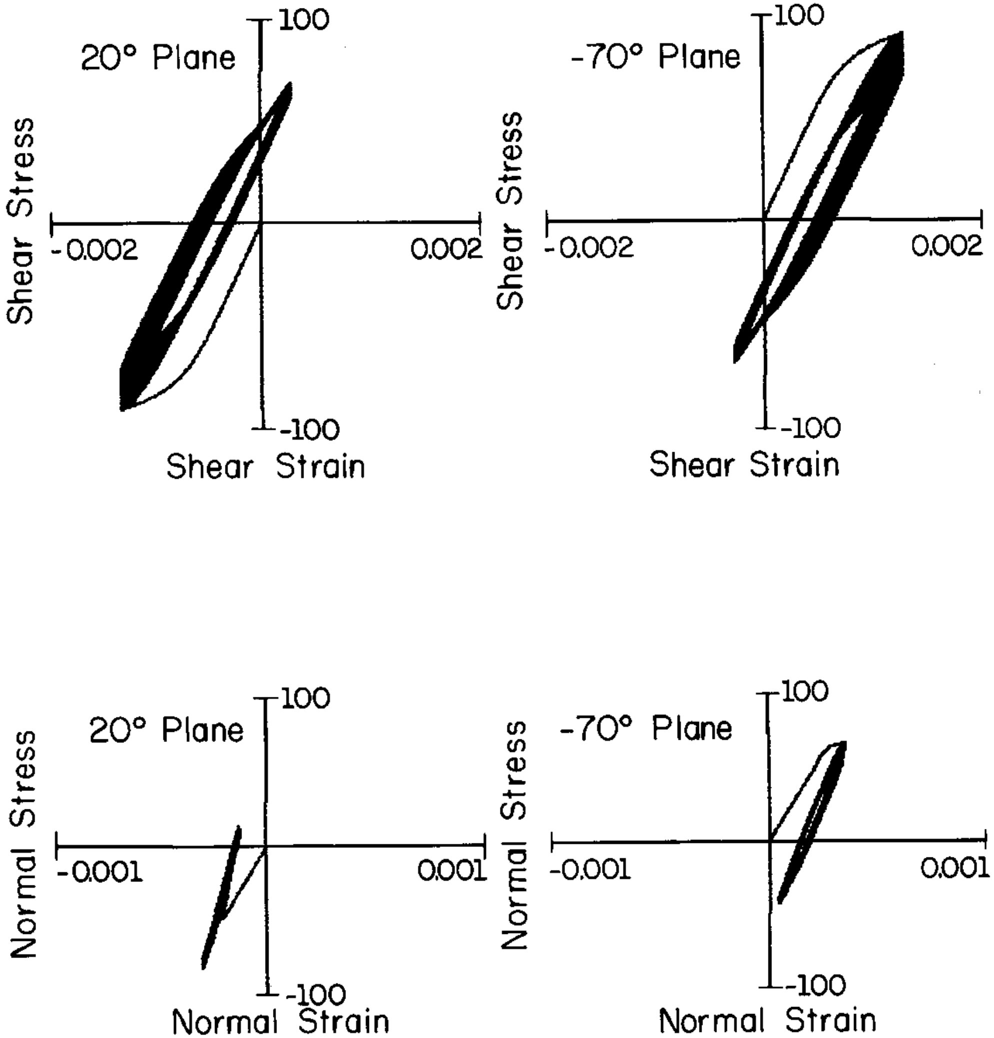
Fatigue damage is planar in nature

The material finds a critical plane for microcrack growth.



Stresses are nearly the same over a 10° range of angles

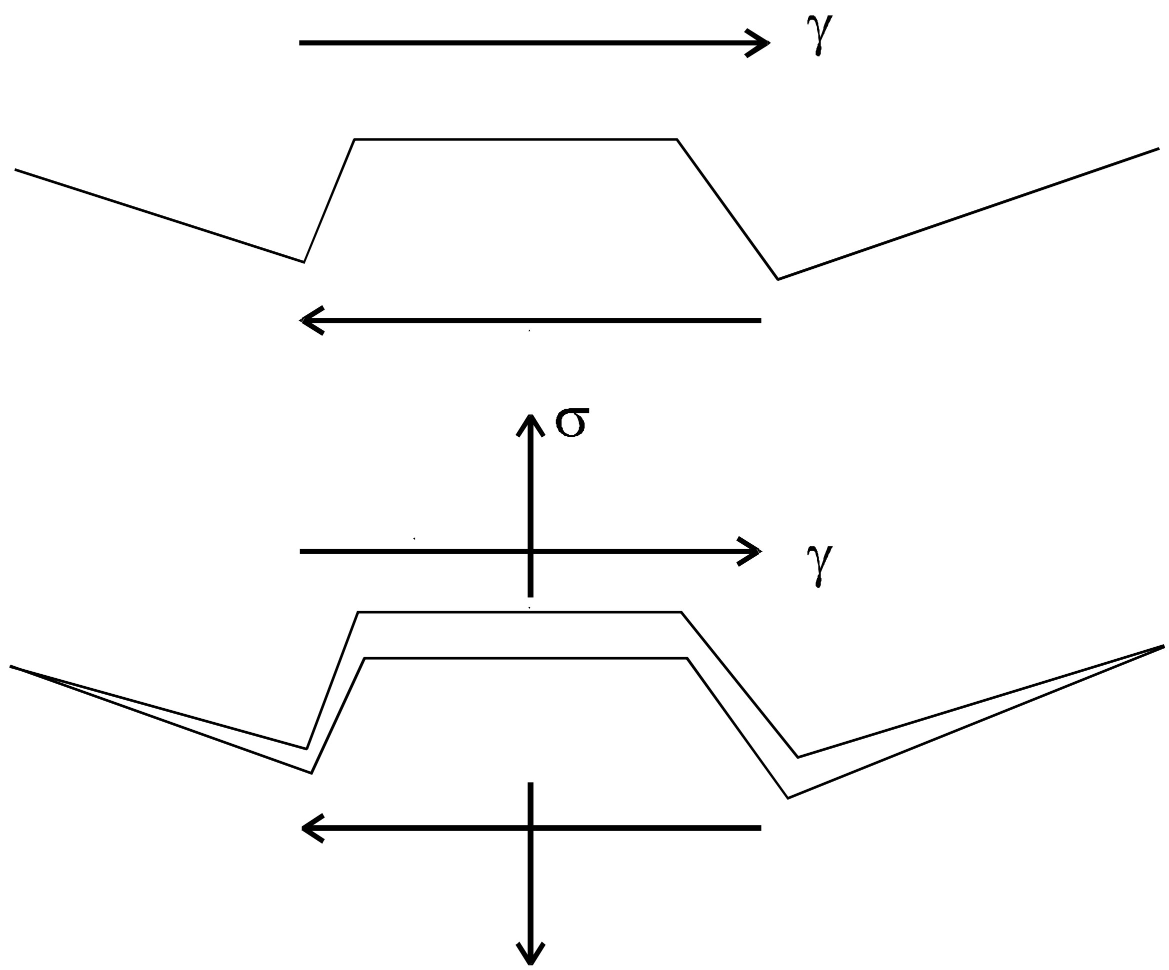
Stresses on the Planes



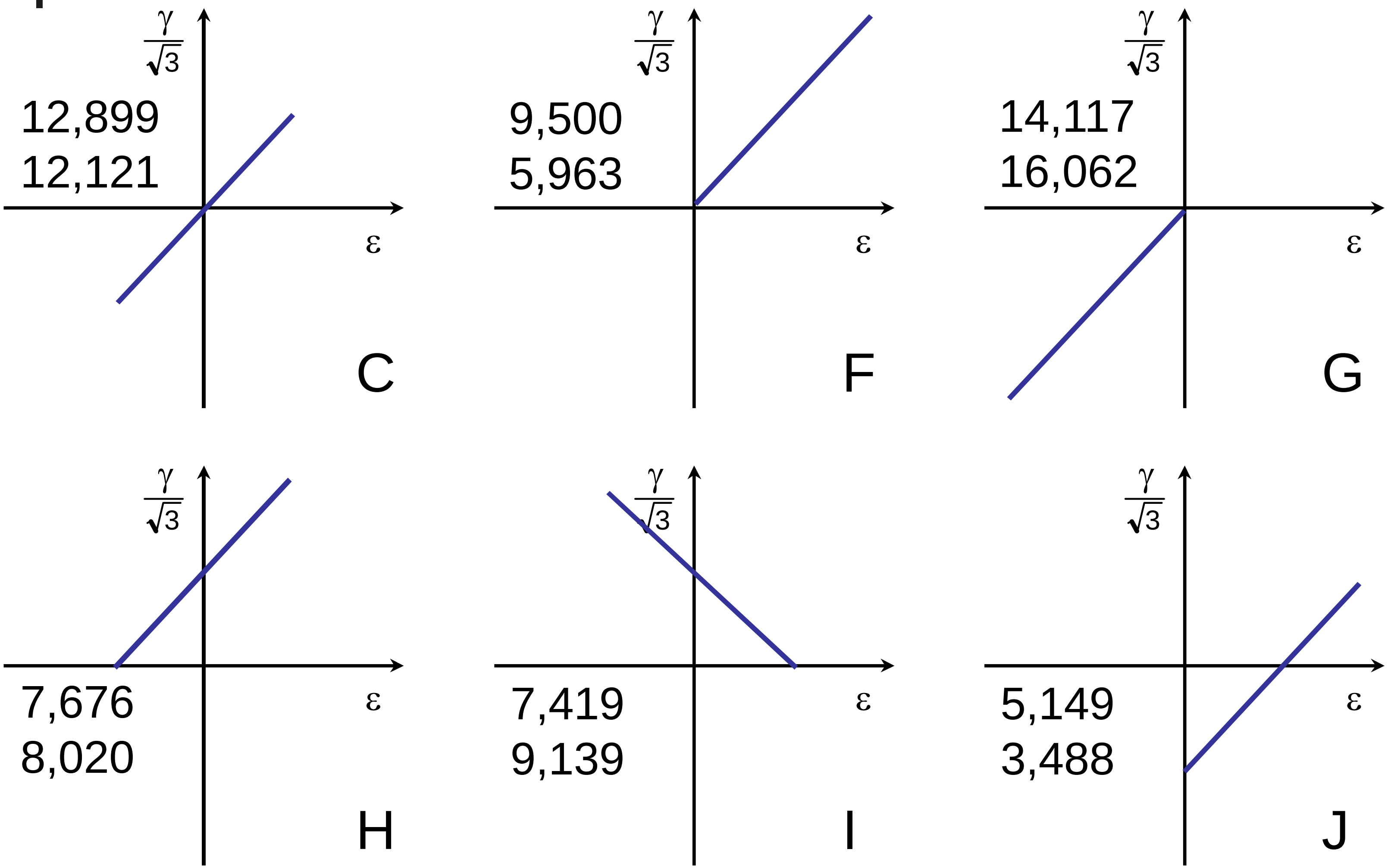
Shear

Tension

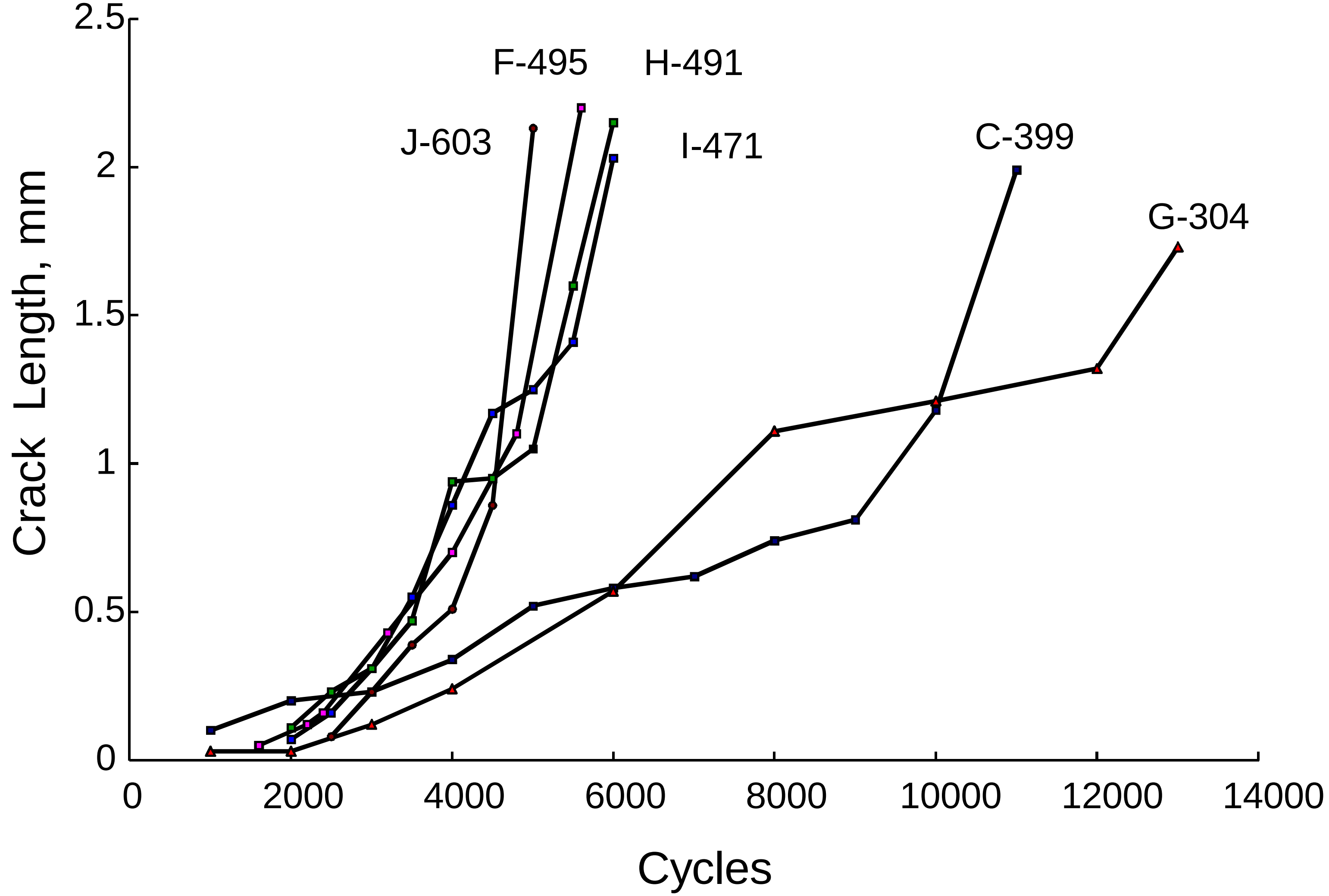
Fatemi and Socie



Fatigue Lives



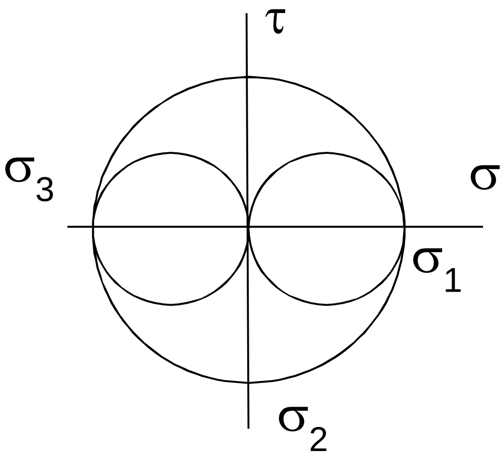
Crack Length Observations



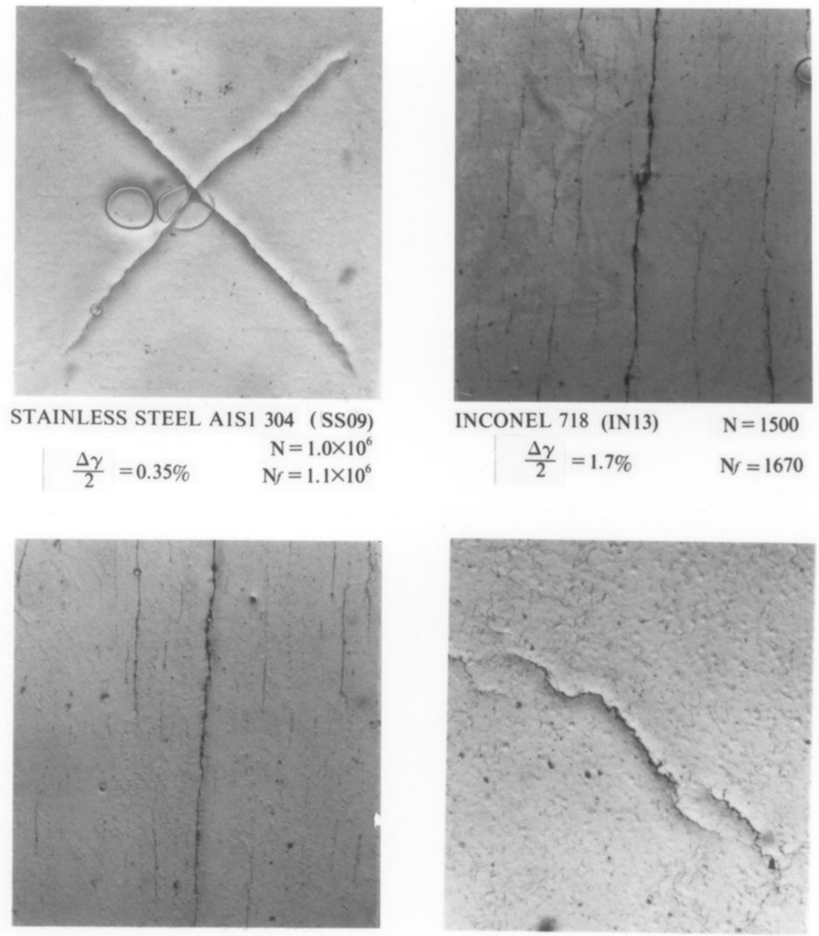
Fatemi and Socie

$$\frac{\Delta\gamma}{2}\left(1+k\frac{\sigma_{n,max}}{\sigma_y}\right)=\frac{\tau_f^{'}}{G}(2N_f)^{bo}+\gamma_f^{'}(2N_f)^{co}$$

Torsion Tests



$$\sigma_1 = \tau_{xy}$$



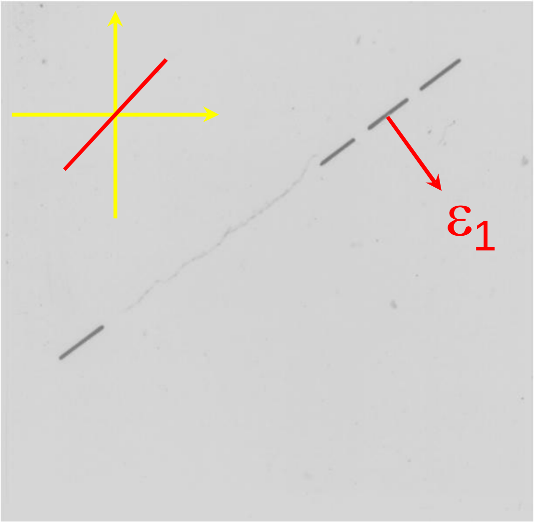
STAINLESS STEEL AISI 304 (SS09) $\frac{\Delta\gamma}{2} = 0.35\%$ $N = 1.0\times10^6$ $N_f = 1.1\times10^6$

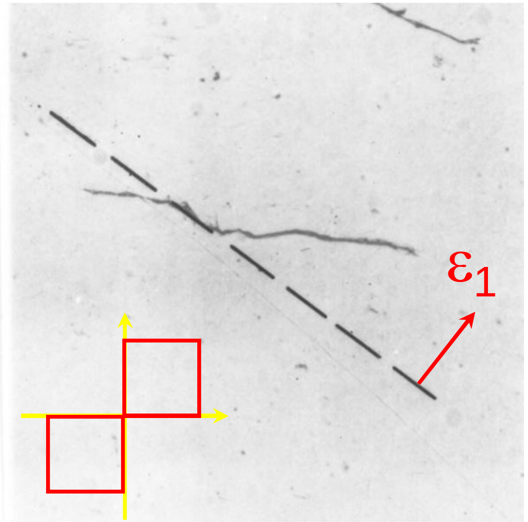
INCONEL 718 (IN13) $\frac{\Delta\gamma}{2} = 1.7\%$ $N = 1500$ $N_f = 1670$

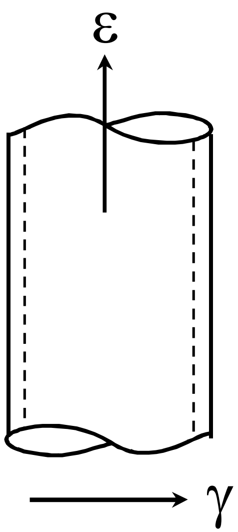
SAE 1045 (4531) $\frac{\Delta\gamma}{2} = 0.38\%$ $N = 90{,}000$ $N_f = 93052$

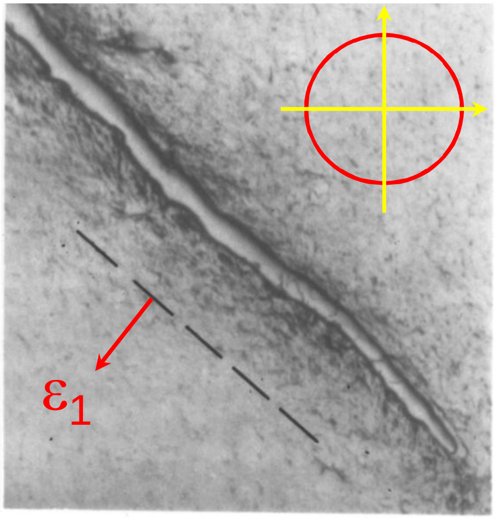
GRAY CAST IRON(NC01) $\frac{\Delta\gamma}{2} = .006$ $N = 400$ $N_f = 440$

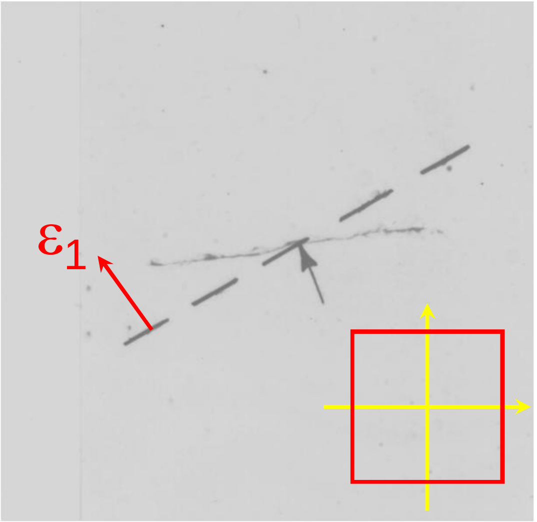
304 Stainless Steel



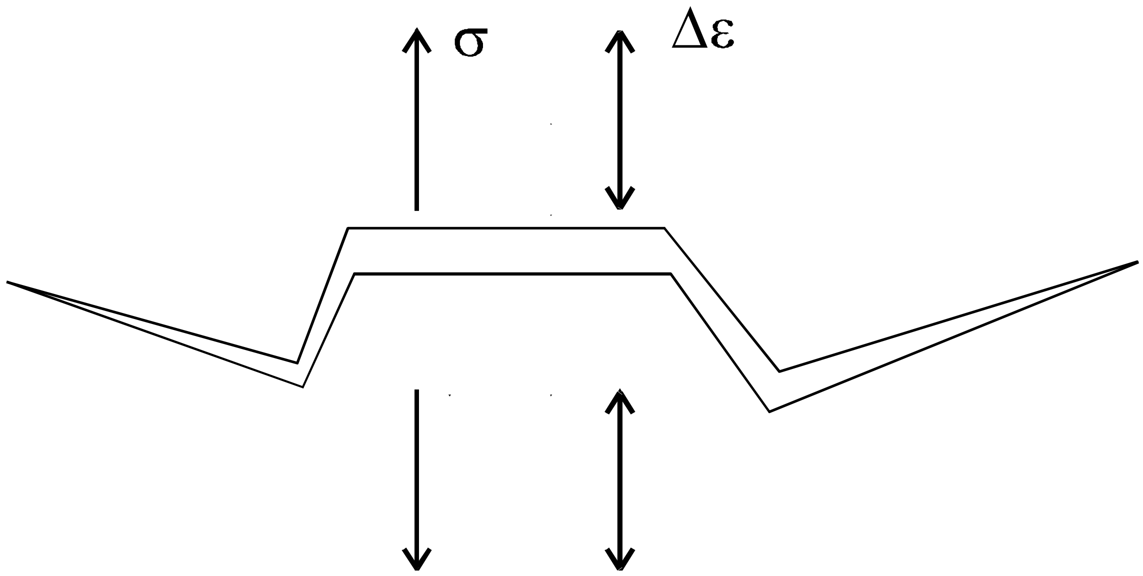








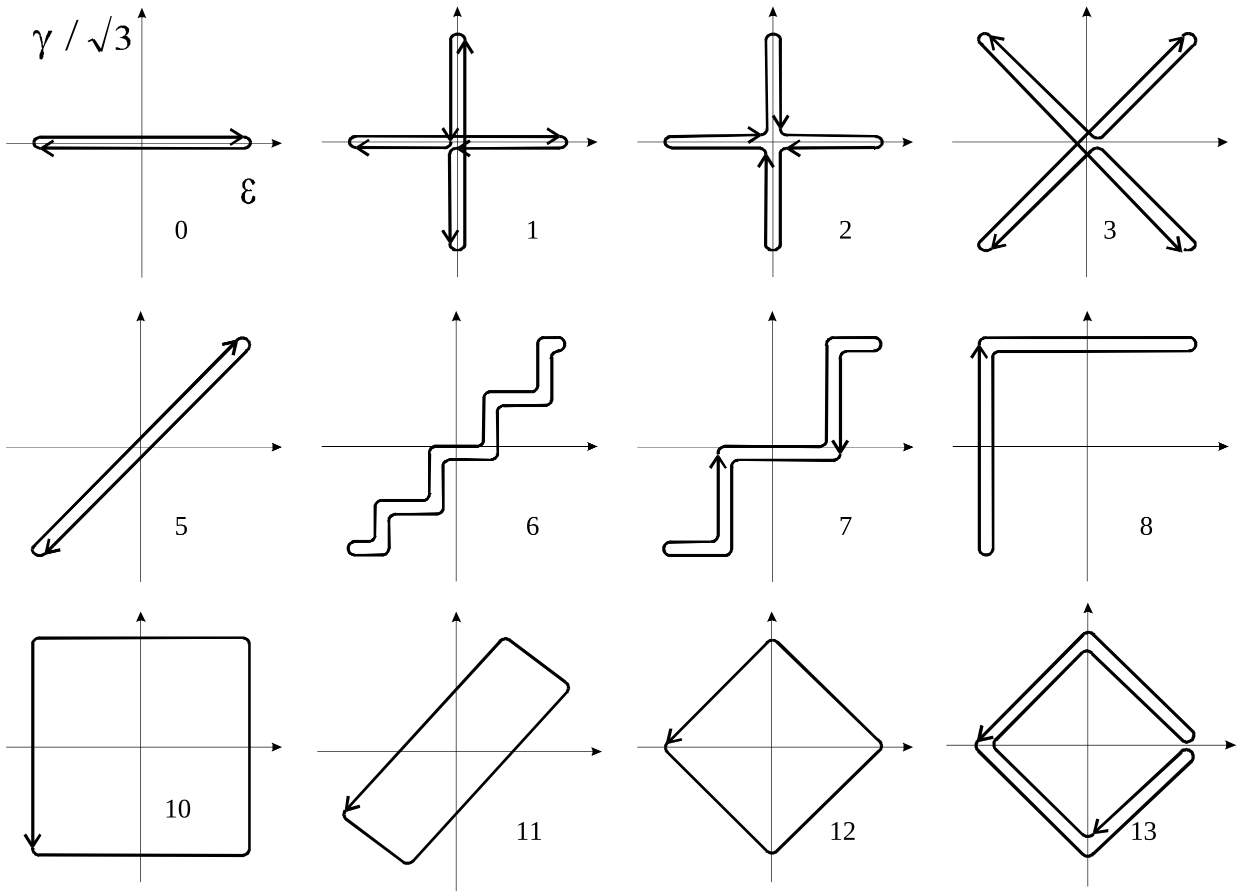
Smith Watson Topper



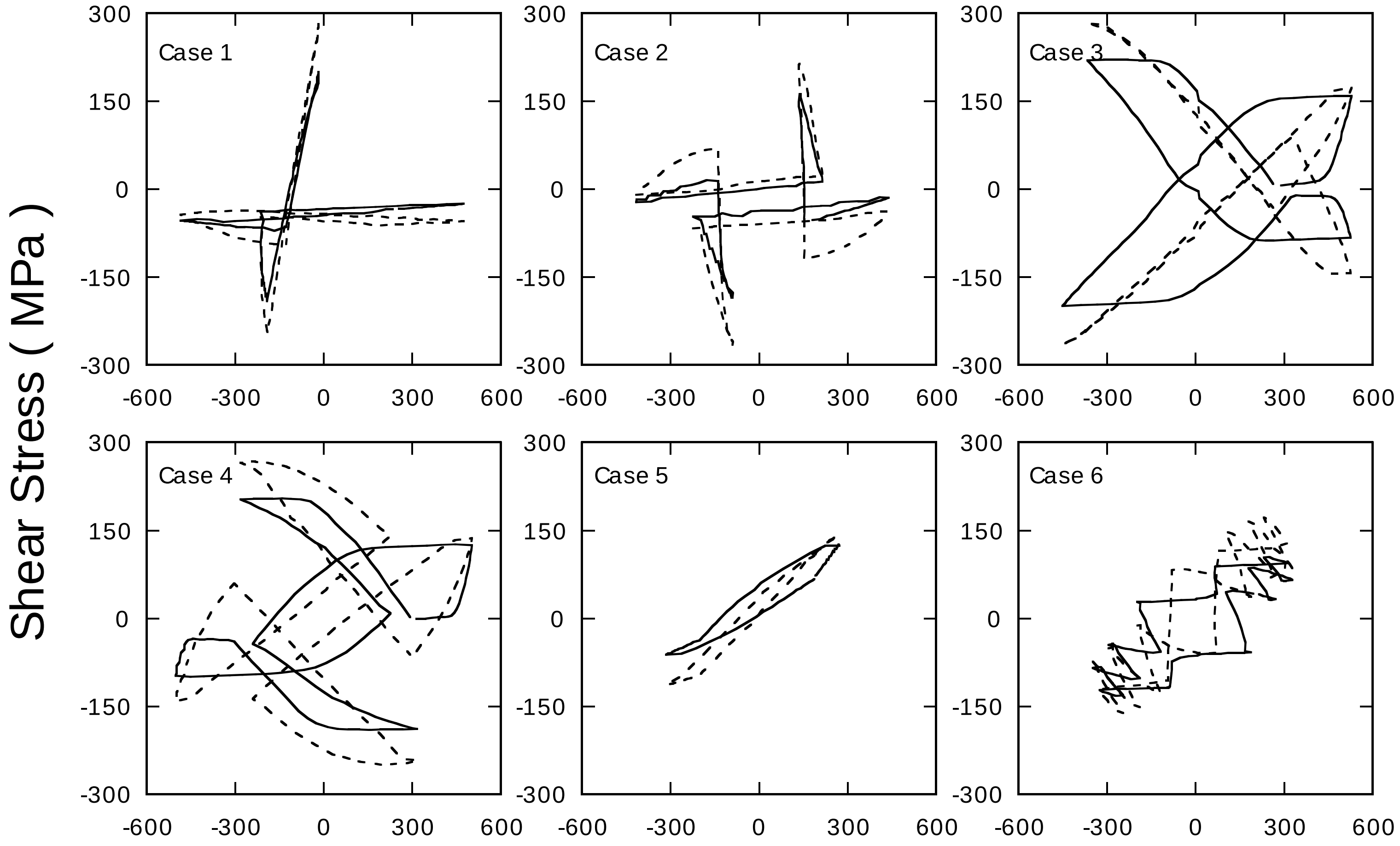
SWT

$$\sigma_n \frac{\Delta\varepsilon_1}{2} = \frac{{\sigma'_f}^2}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c}$$

Loading Histories

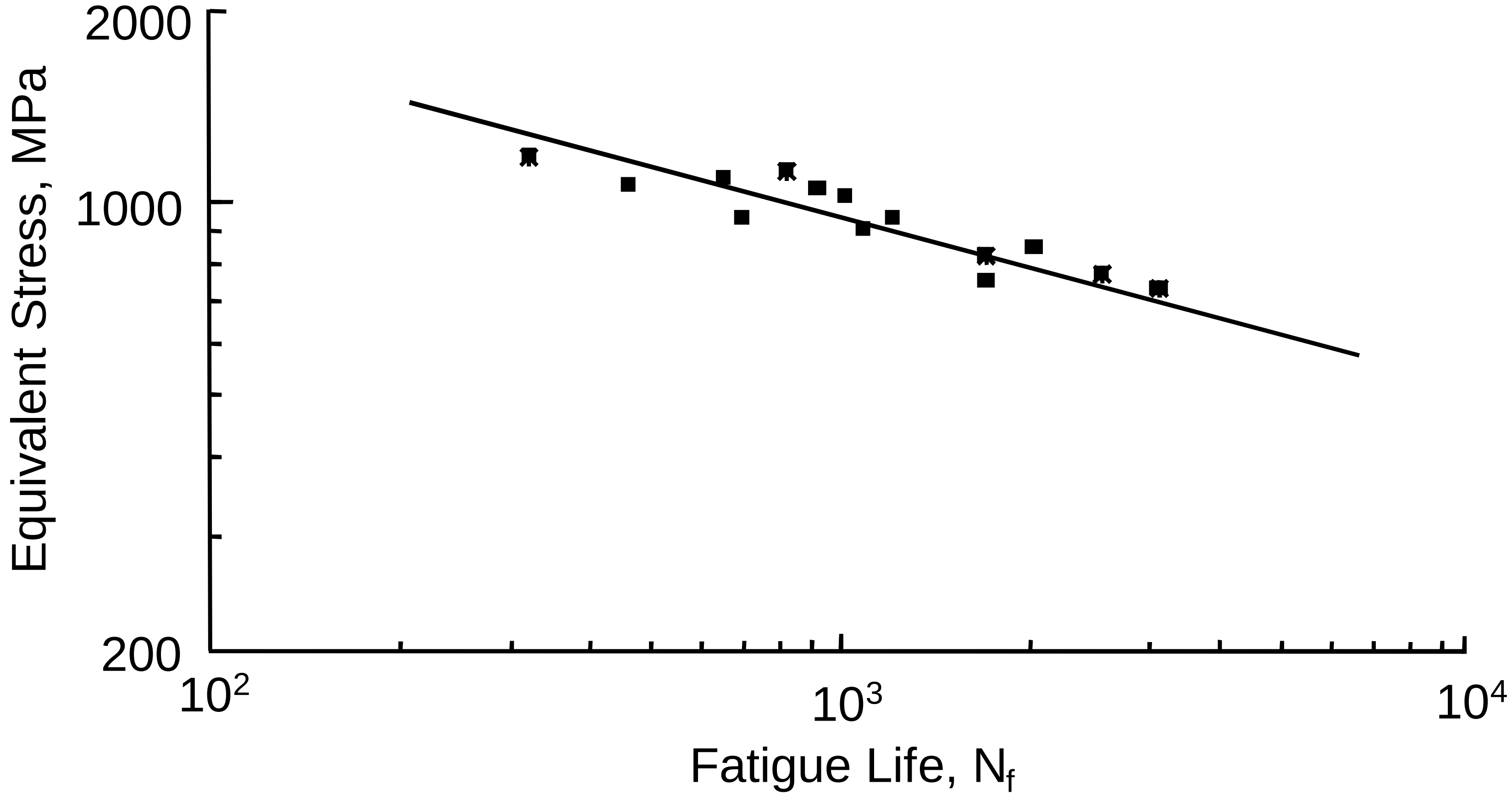


Stress-Strain Response



Maximum Stress

All tests have the same strain ranges



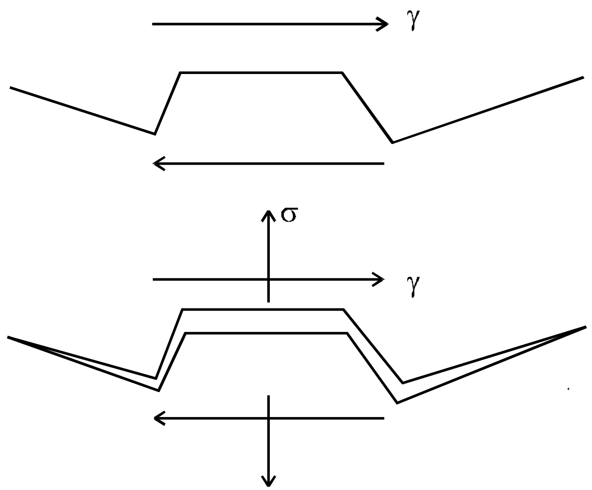
Nonproportional hardening results in lower fatigue lives

Summary

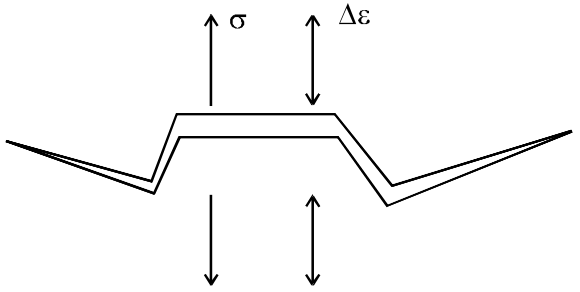
Cracks nucleate in shear and then grow in either shear or tension depending on the material and state of stress

Separate Tensile and Shear Models

Cyclic shear strains

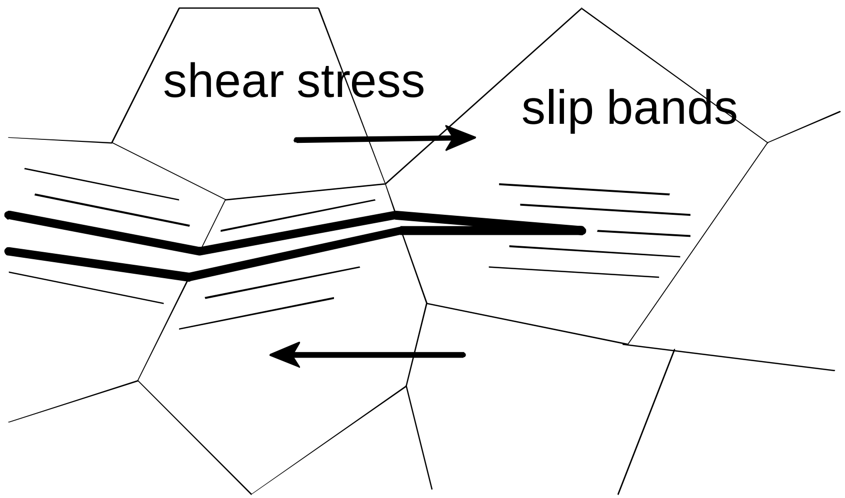


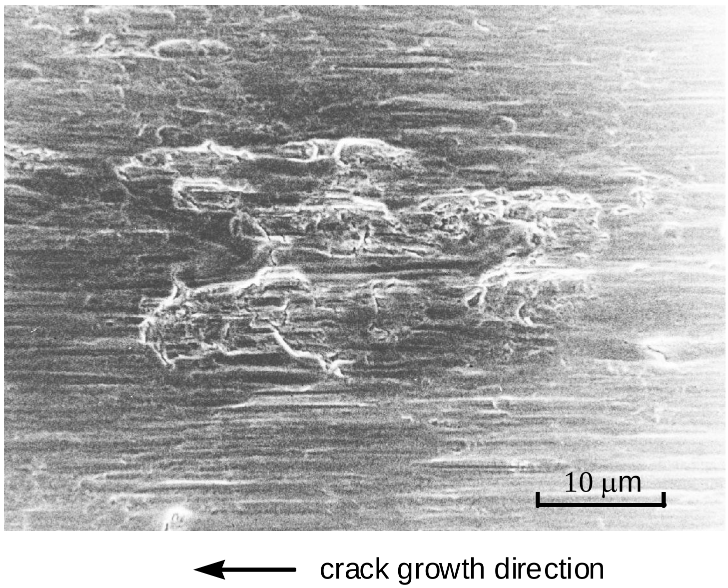
Cyclic tensile strains



Normal stresses open and close microcracks

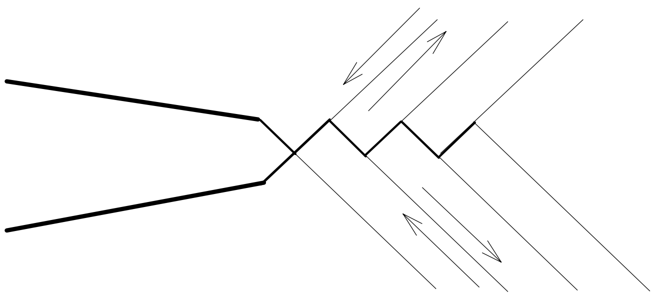
Shear Growth

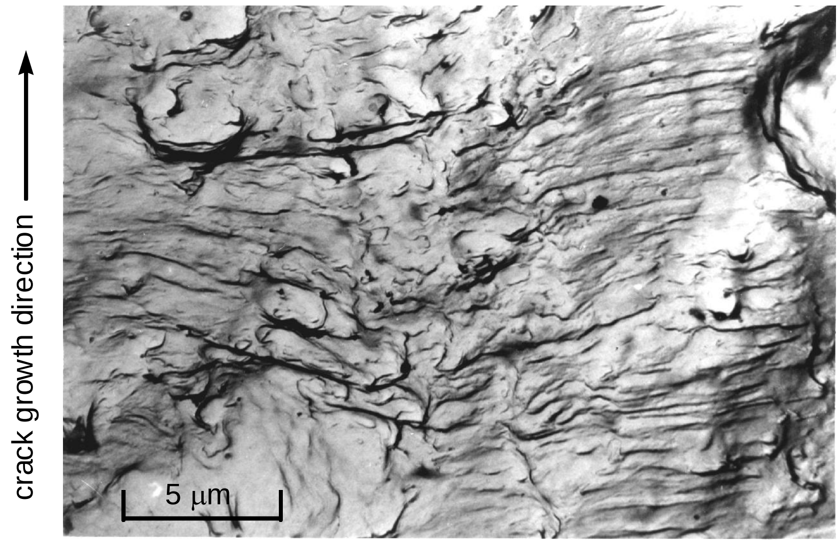




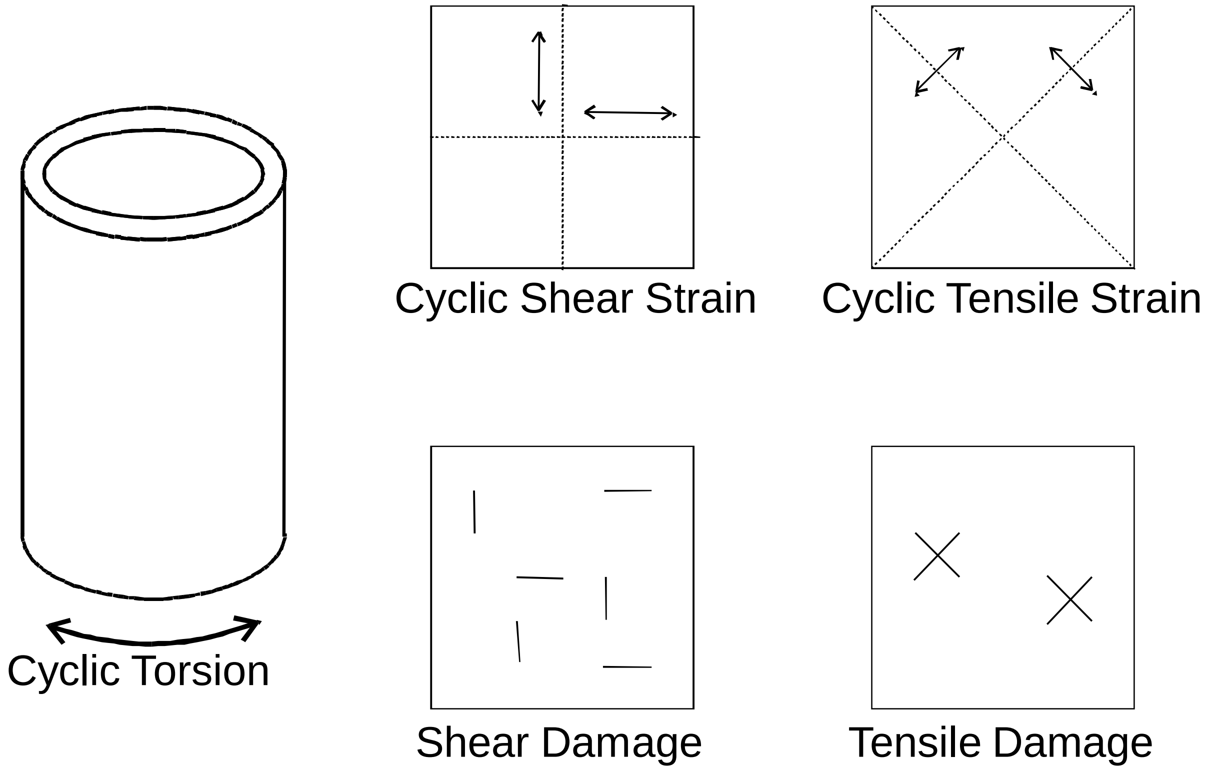
(From Murakami)

Tensile Growth

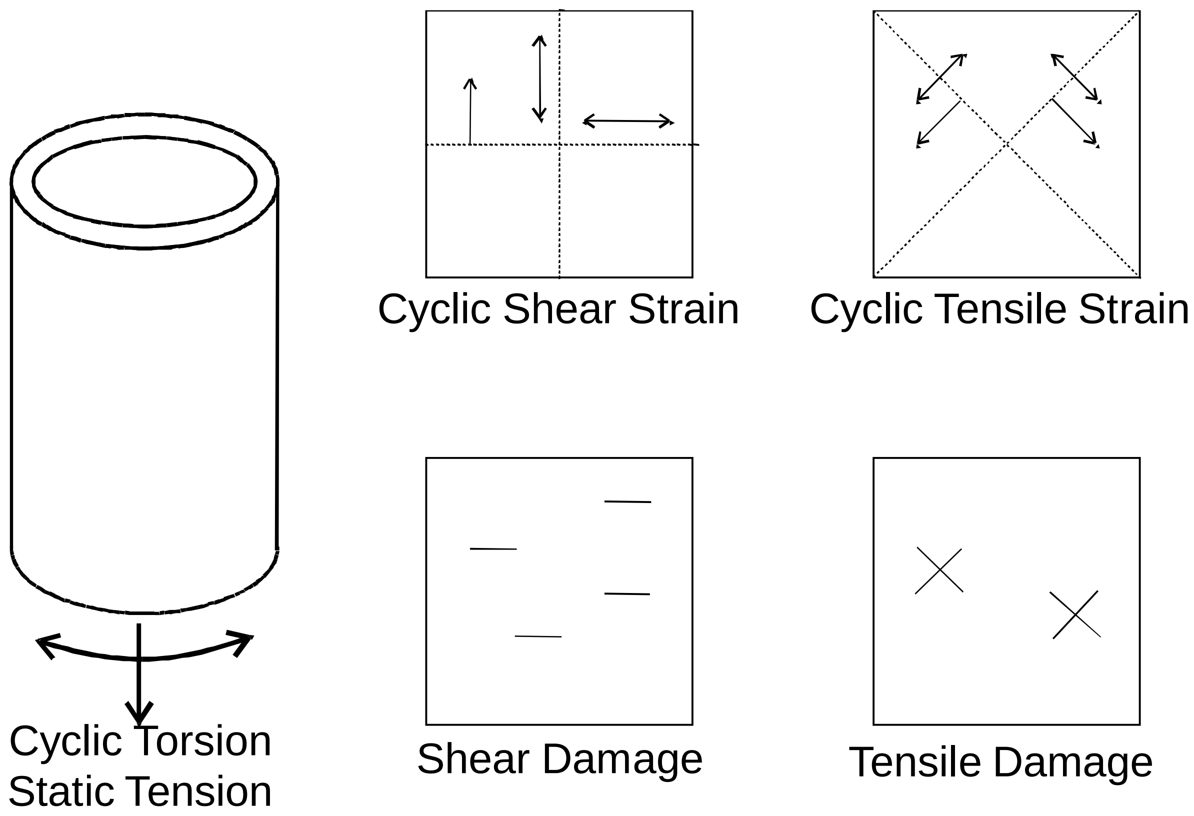




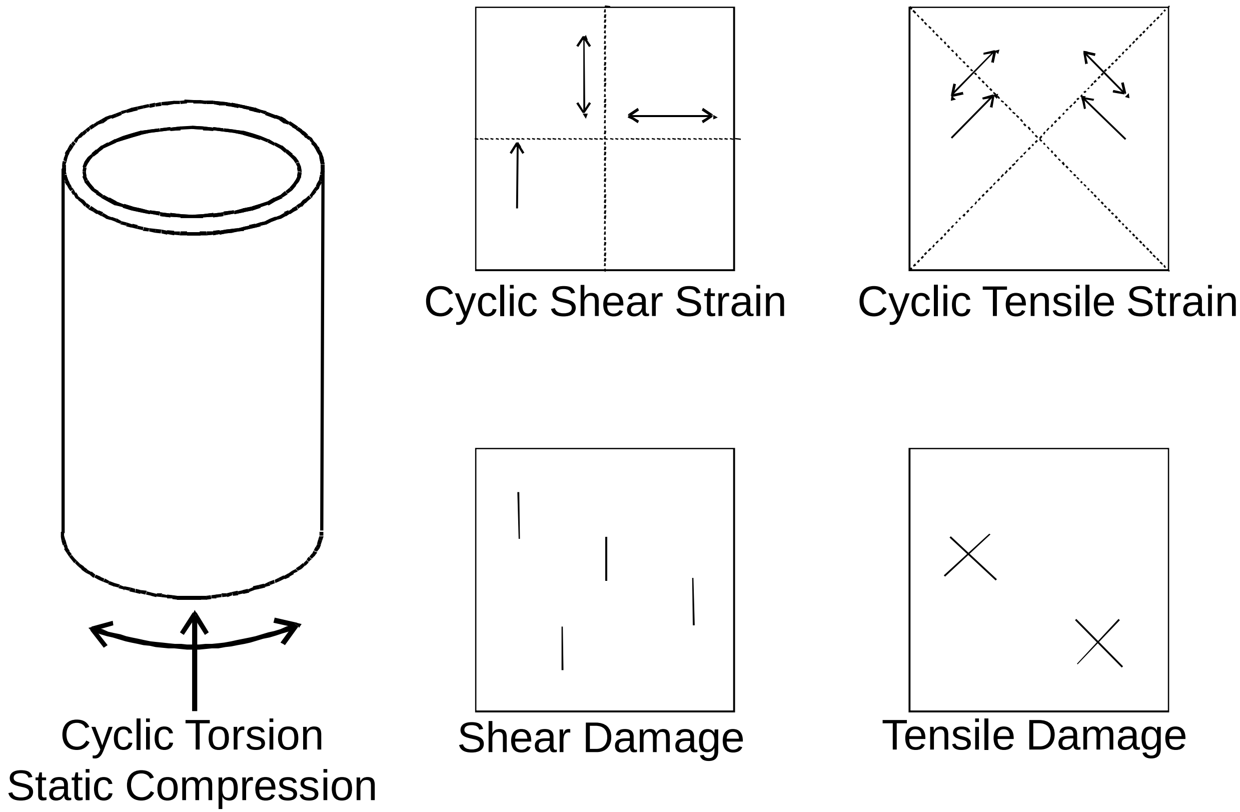
Cyclic Torsion



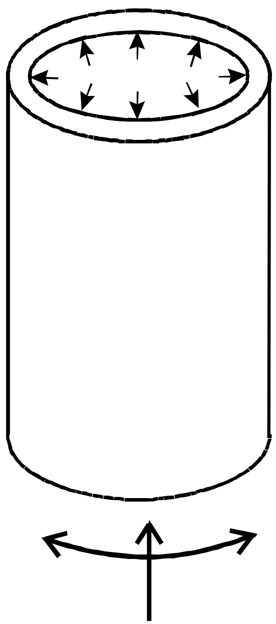
Cyclic Torsion with Static Tension



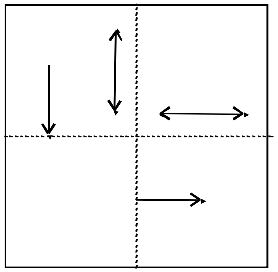
Cyclic Torsion with Compression



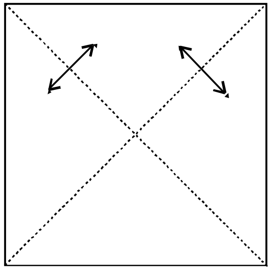
Cyclic Torsion with Tension and Compression



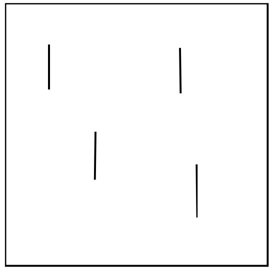
Cyclic Torsion
Static Compression
Hoop Tension



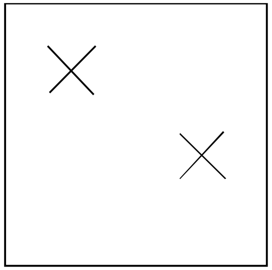
Cyclic Shear Strain



Cyclic Tensile Strain



Shear Damage



Tensile Damage

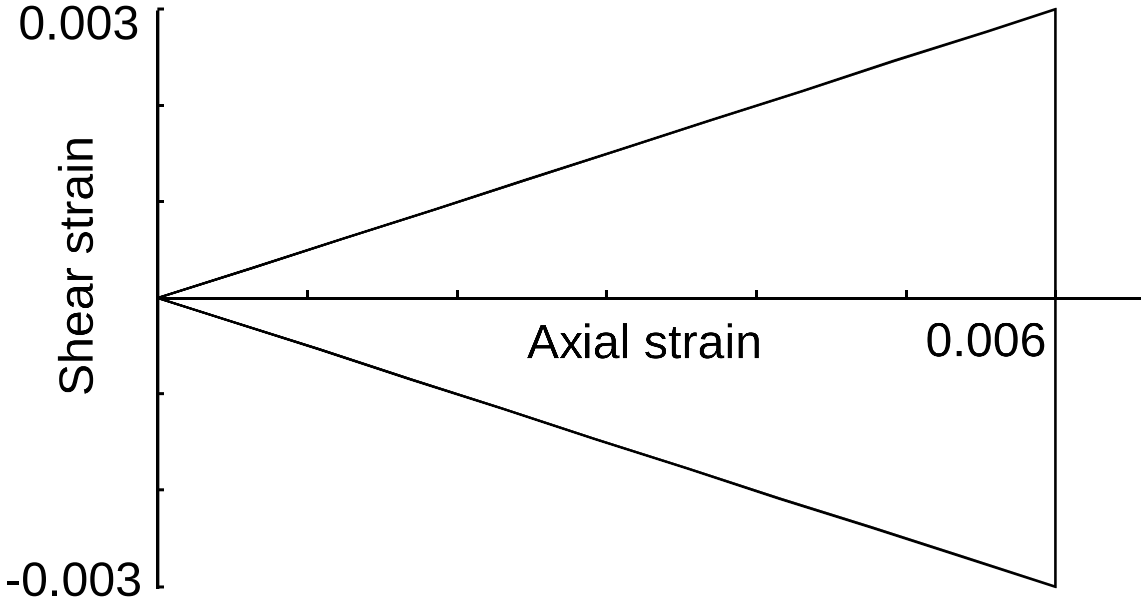
Test Results

Load Case	$\Delta\gamma/2$	σ_{hoop} MPa	σ_{axial} MPa	N_f
Torsion	0.0054	0	0	45,200
with tension	0.0054	0	450	10,300
with compression	0.0054	0	-500	50,000
with tension and compression	0.0054	450	-500	11,200

Conclusions

- All critical plane models correctly predict these results
- Hydrostatic stress models can not predict these results

Loading History



Model Comparison

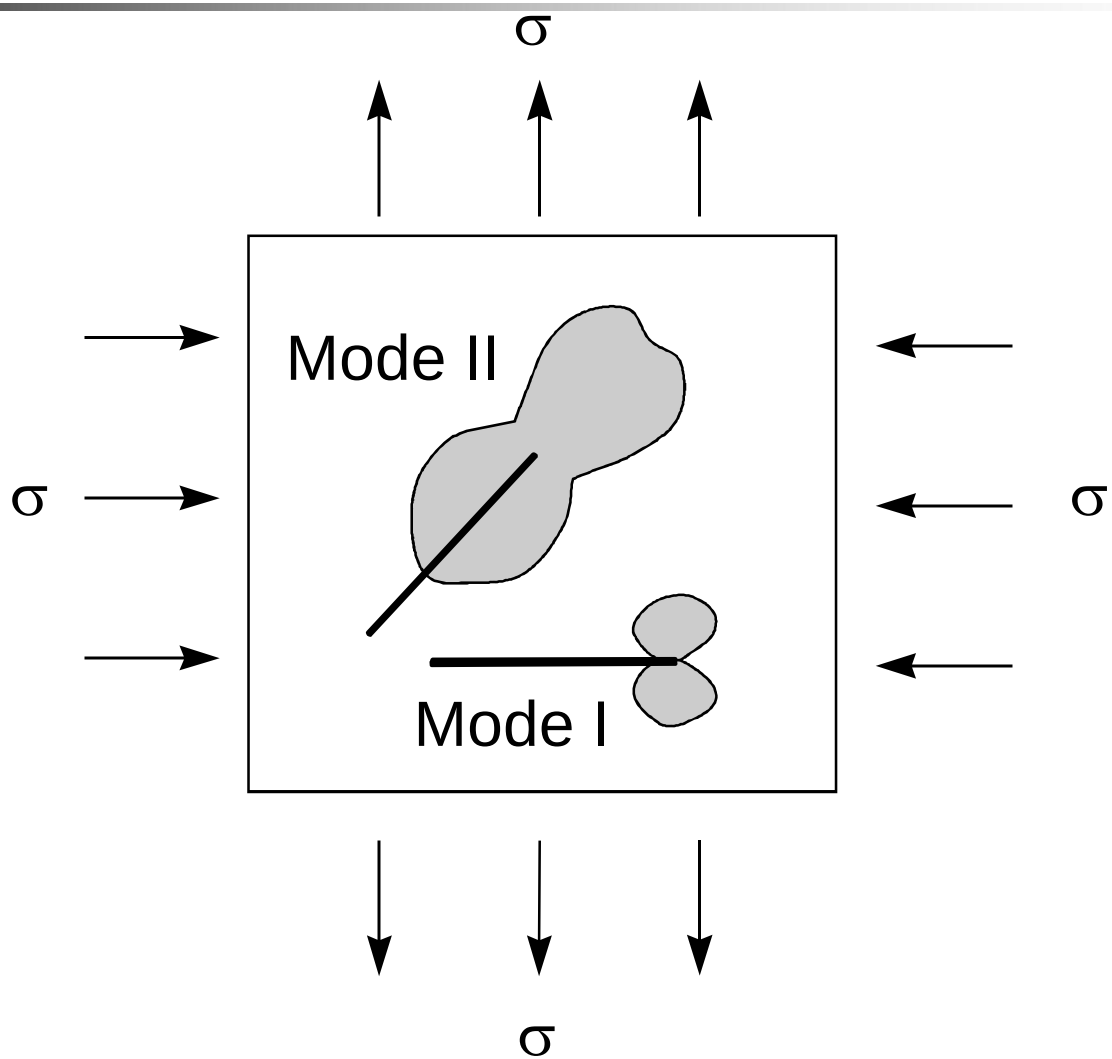
Summary of calculated fatigue lives

Model	Equation	Life
Epsilon	6.5	14,060
Garud	6.7	5,210
Ellyin	6.17	4,450
Brown-Miller	6.22	3,980
SWT	6.24	9,930
Liu I	6.41	4,280
Liu II	6.42	5,420
Chu	6.37	3,040
Gamma		26,775
Fatemi-Socie	6.23	10,350
Glinka	6.39	33,220

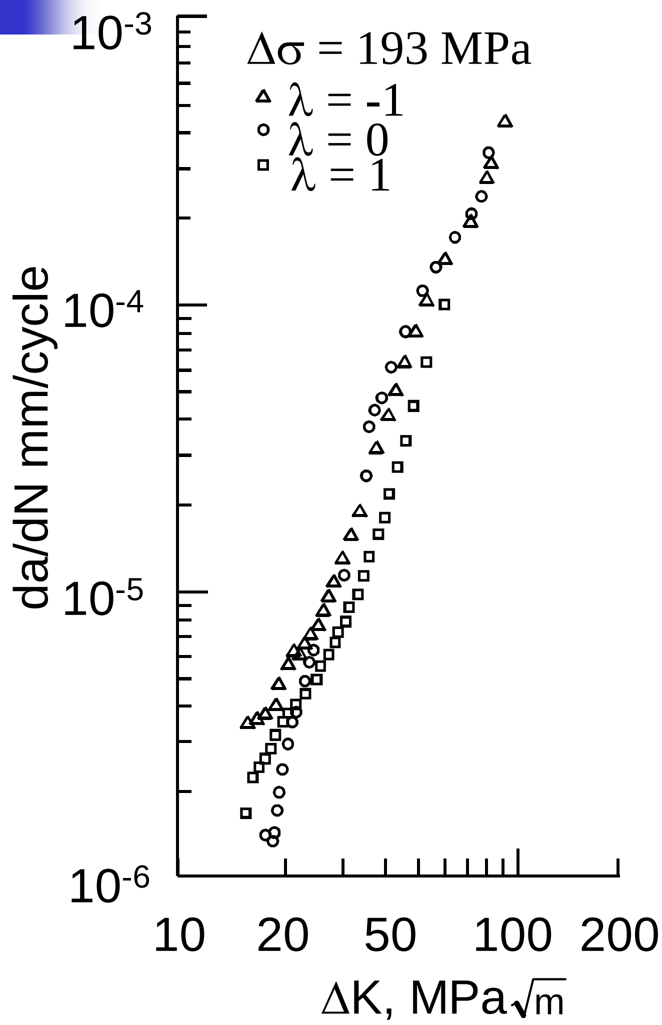
Outline

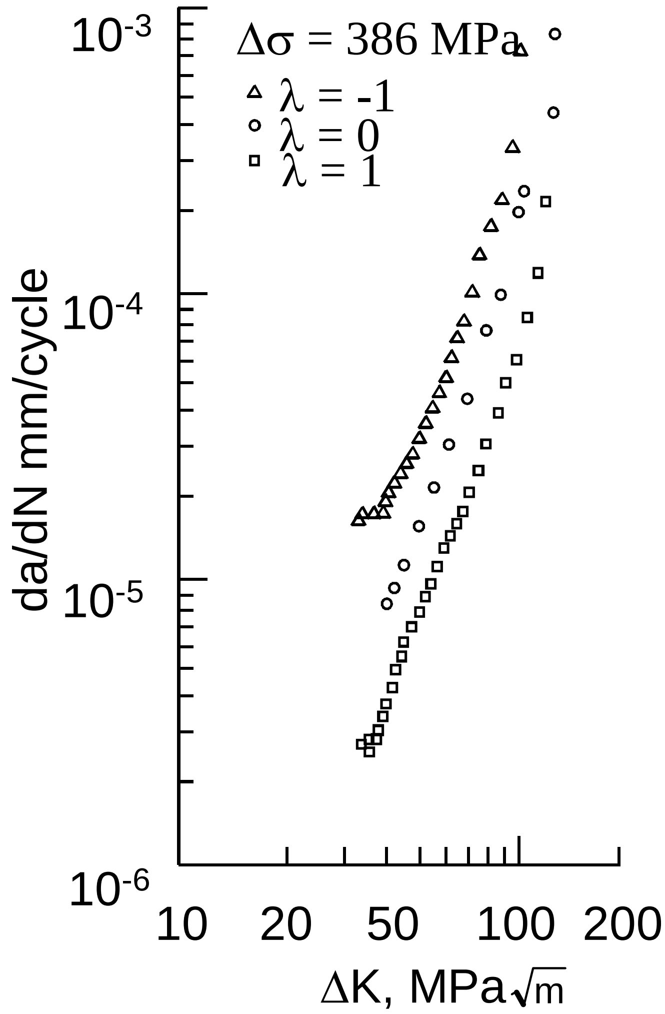
- Stresses around holes
- Crack Nucleation
- Crack Growth

Mode I and Mode II Surface Cracks

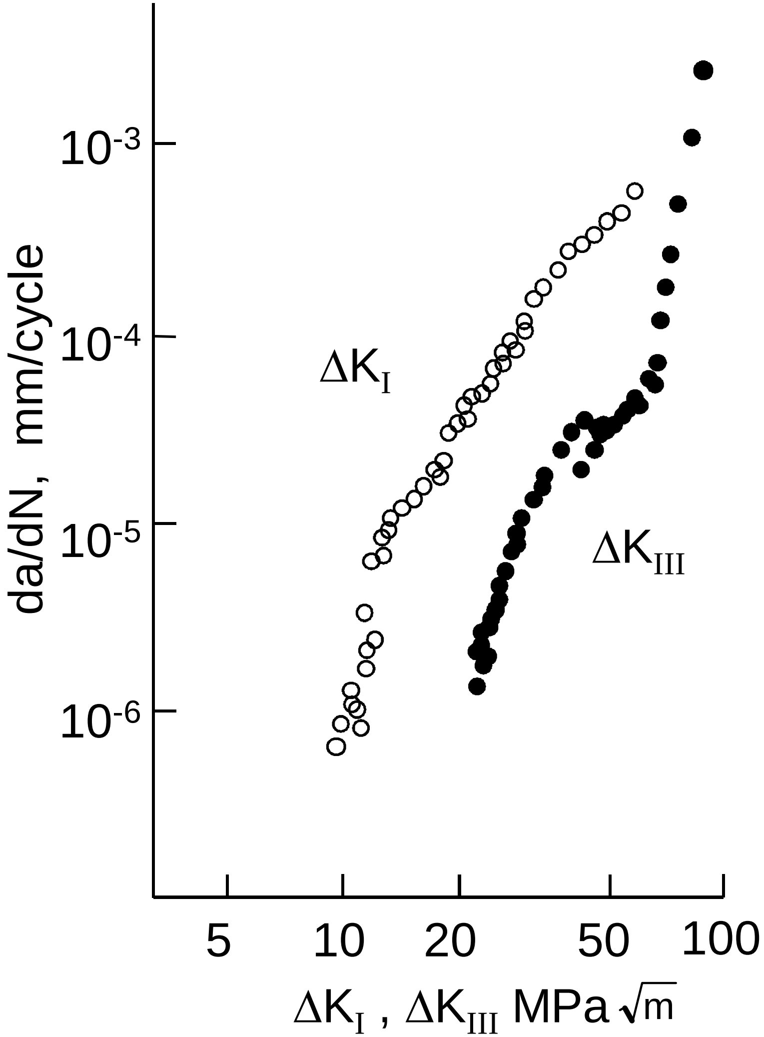


Biaxial Mode I Growth

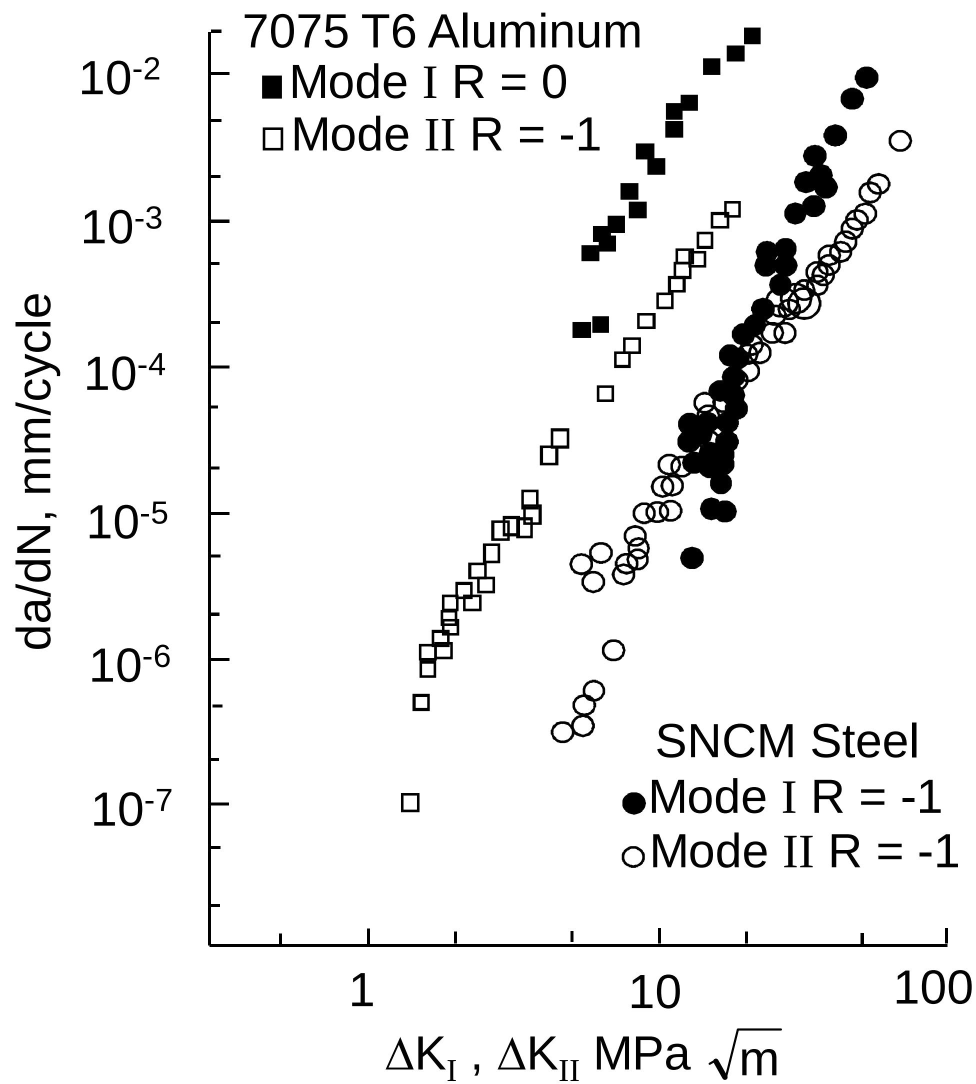




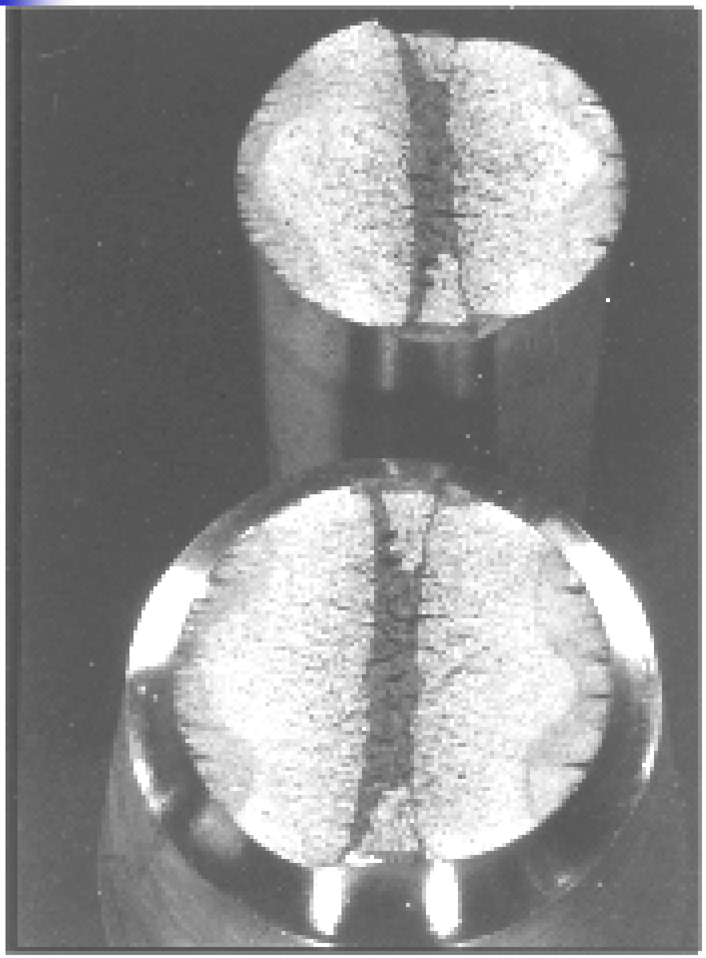
Mode I and Mode III Growth



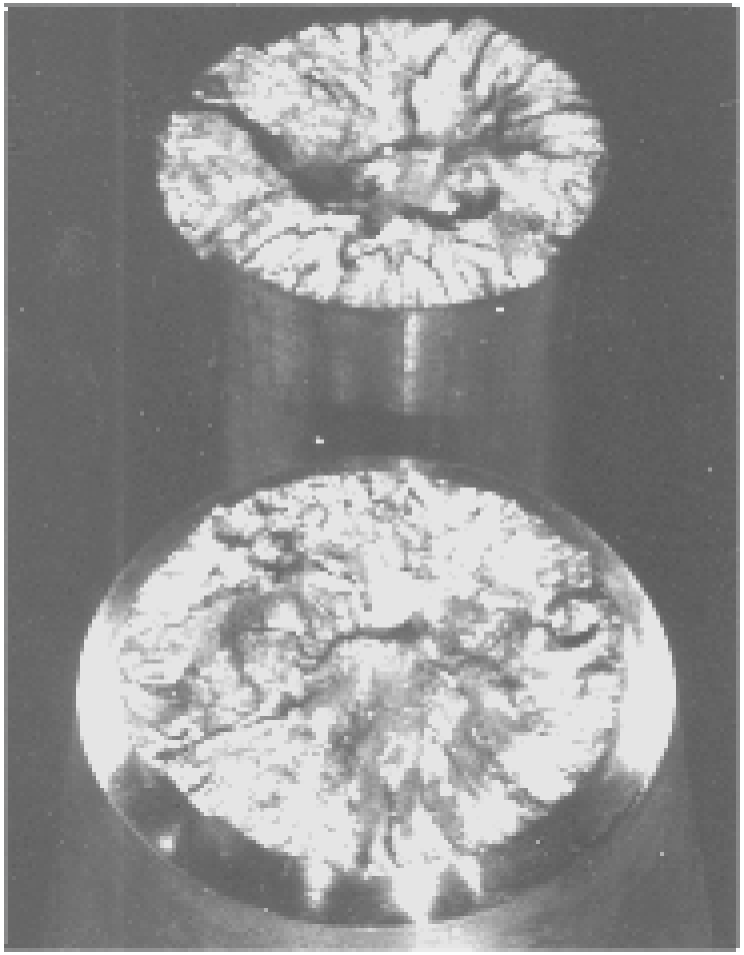
Mode I and Mode II Growth



Fracture Surfaces

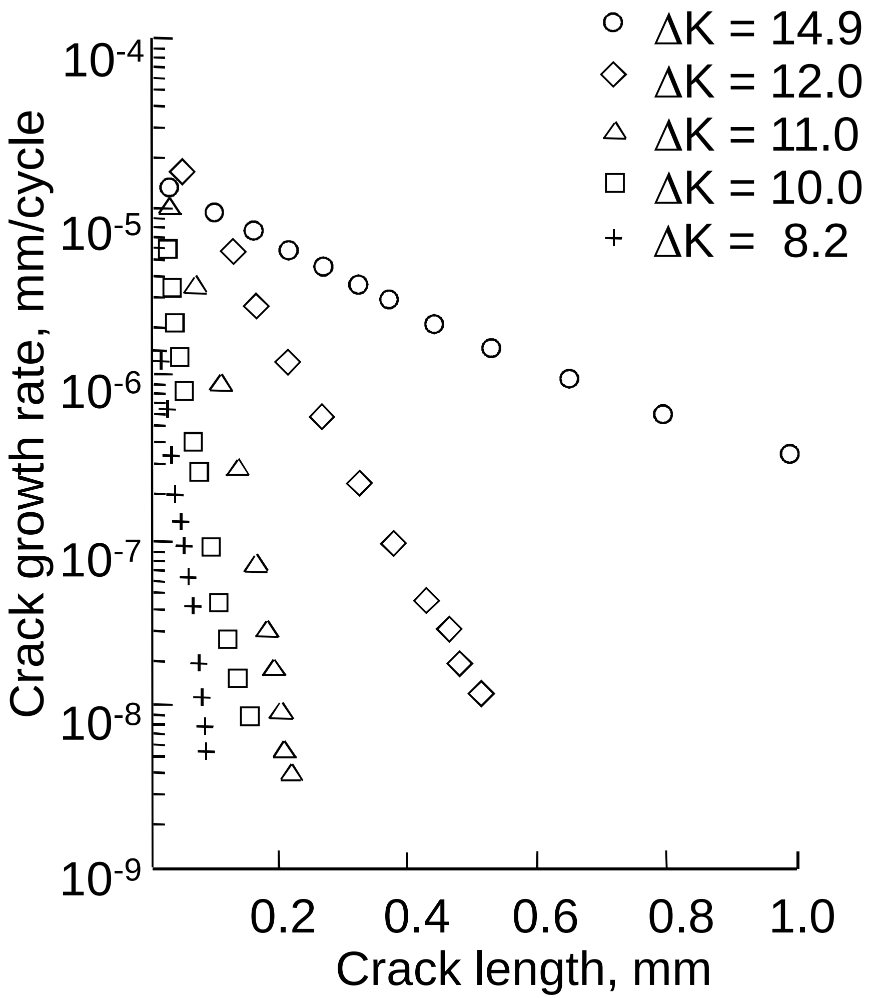


Bending



Torsion

Mode III Growth



Fracture Mechanics Models

$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m$$

$$\Delta K_{eq} = \left[\Delta K_I^4 + 8\Delta K_{II}^4 + 8\Delta K_{III}^4 / (1-\nu)\right]^{0.25}$$

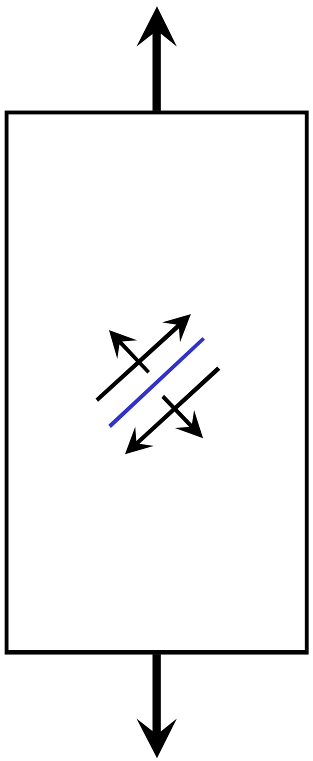
$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_{II}^2 + (1+\nu)\Delta K_{III}^2\right]^{0.5}$$

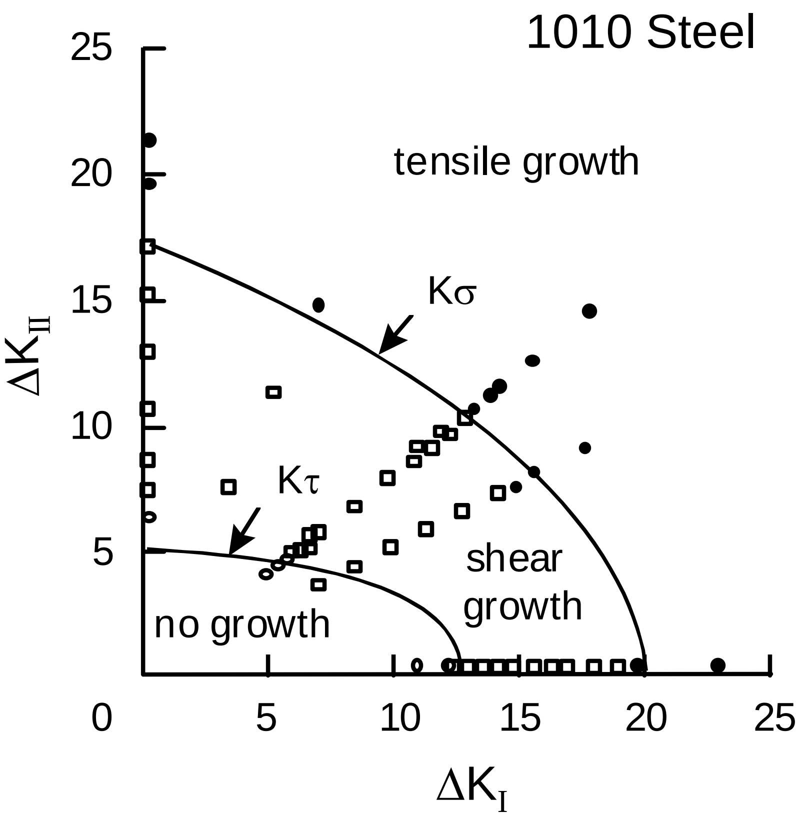
$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_I \, \Delta K_{II} + \Delta K_{II}^2\right]^{0.5}$$

$$\Delta K_{eq}(\varepsilon) = \left[\left(F_{II}\frac{E}{2(1+\nu)}\Delta\gamma\right)^2 + \left(F_I E \Delta\varepsilon\right)^2\right]^{0.5} \sqrt{\pi a}$$

$$\Delta K_{eq}(\varepsilon) = FG\Delta\gamma\left(1 + k\frac{\sigma_{n,max}}{\sigma_{ys}}\right)\sqrt{\pi a}$$

Growth of Inclined Cracks





Cracks grow in either tension or shear

From: Otsuka et.al. *Engineering Fracture Mechanics*, Vol 7, 1975

Otsuka

Tensile growth:

$$K\sigma = \cos\frac{\theta}{2}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right]$$

Shear growth:

$$K\tau = \frac{1}{2}\cos\frac{\theta}{2}\left[K_I \sin\frac{\theta}{2} + K_{II}(3\cos\theta - 1)\right]$$

Strain Energy Density

Strain energy density at the crack tip:

$$S = a_{11}K_I^2 + 2a_{12}K_IK_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2$$

Necessary and sufficient conditions for crack growth:

$$\frac{\partial S}{\partial \theta} = 0 \quad \text{at } \theta = \theta_o$$

$$\frac{\partial^2 S}{\partial \theta^2} > 0 \quad \text{at } \theta = \theta_o$$

Cyclic strain energy density:

$$\Delta S = 2[a_{11}(\theta_o)K_I^{mean}\Delta K_I + a_{12}(\theta_o)(K_{II}^{mean}\Delta K_I + K_I^{mean}\Delta K_{II})$$
$$+ a_{22}(\theta_o)K_{II}^{mean}\Delta K_{II} + a_{33}(\theta_o)K_{III}^{mean}\Delta K_{III}]$$

Sih, G.C and Barthelemy, B.M. "Mixed Mode Fatigue Crack Growth Predictions" *Engineering Fracture Mechanics*, Vol. 13, 1980

Summary

Many models but no experimental verification for out-of-phase spectrum loads

Multiaxial Fatigue